EXCITATION OF AN ATOM BY A NONMONOCHROMATIC ELECTROMAGNETIC FIELD

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The time evolution of the process of excitation of an atom in the field of a laser wave is considered. It is shown that there exist three stages of relaxation. The first stage carries the most complete information about the mechanism of the interaction of the field with material inside the laser source.

In connection with the production of high-power sources of optical radiation, much attention has been given in the literature of recent years to the investigation of the behavior of atomic systems in nonmonochromatic fields [1-8]. Nonlinear effects have been taken into account. The statistical nature of the radiation, as a rule, has been determined by the phenomenological equality

\[ \langle \varepsilon(0)\varepsilon(t) \rangle = \varepsilon^2 \exp \left( -\Gamma |t| \right). \]  

We will make use of a fundamentally different method in this work. We will assume that the radiation is formed by a laser system describable in the spirit of the Scully–Lamb theory [9]. It is found that not all the statistical properties of the radiation are encompassed by Eq. (1). In spite of the fact that the proposed formalism permits studying nonlinear effects, we will not consider this problem here, while drawing attention only to the different stages of the relaxation process. That is, we will be interested in the time evolution of the process of excitation of an atom, which in turn permits judging the spectral width of the laser radiation and possible phenomena in nonlinear processes. Moreover, such a formulation is of obvious interest for the theory of photoreadings.

Let us consider a single-electron "test" atom in a transverse electromagnetic field generated by a medium also consisting of single-electron atoms. Under sufficiently standard physical assumptions the wave function of this system in the second quantization representation satisfies the equation

\[ i \frac{\partial \Psi}{\partial t} = \left[ \hat{H}_{ph} + \hat{H}_a + \hat{H}_p - \frac{\hat{p}^2}{2m} \hat{\lambda}\hat{\lambda} - \frac{\varepsilon}{m} \int \hat{\phi}^+(r, R) \hat{\phi}^+(r, R) \right. \]

\[ \left. \times \hat{\phi}^+(r, R) \hat{\phi}^+(r, R) dr dR \right] \Psi. \]

Here

\[ \hat{H}_{ph} = \sum_{kk'} \left( \hat{a}_{k}\hat{a}_{k'}^{\dagger} + \frac{1}{2} \right), \quad \hat{H}_a = \sum_{jp} \hat{b}_{jp}^{\dagger} \hat{b}_{jp} \hat{\phi}, \quad \hat{p}_r = -i \varepsilon^2. \]

\[ \hat{\lambda}\hat{\lambda} = \sum_{kk'} \frac{\varepsilon_{k}}{\sqrt{2\varepsilon_{k}}} \hat{a}_{k}\hat{a}_{k'}^{\dagger}, \quad \hat{H}_p = -\frac{\hat{p}^2}{2m} + \]

\[ \times \hat{V}(\hat{p}), \quad \varepsilon_{jp} = \varepsilon_j + \frac{\hat{p}^2}{2M}. \]

The argument \( \hat{p} \) characterizes the bound electron in the "test" atom,

\[ \hat{\phi}(r, R) = \sum_{jp} \hat{b}_{jp} \hat{\phi}_j(r - R) \frac{\exp ipr}{\sqrt{V}}. \]

where \( \psi_j \) is the wave function of the atoms of the medium, which have internal energy \( \varepsilon_j \). We will neglect the width of the energy level. The operators \( \hat{b}^+_j, \hat{a}^+_k(\hat{b}_j, \hat{a}_k) \) are the creation (annihilation) operators of the atoms and photons. The statistics of the atoms of the medium play no role in the absence of temperature degeneracy. One can thus assume

\[
[\hat{a}_{k\lambda} \hat{a}_{k'\lambda}^+] = \delta_{kk'} \delta(\mathbf{K} - \mathbf{K}'), [\hat{b}_{j\mu}, \hat{b}_{j'\mu}^+] = \delta_{jj'} \delta(\mathbf{p} - \mathbf{p}').
\]

The charge \( q \) coincides, generally speaking, with the electron charge \( e \), but we will assume it to be time dependent. Summation over the index \( \lambda \) is understood here and below. Let

\[
\hat{\zeta}_{k\lambda} = \frac{1}{\sqrt{2}} \left( \zeta_{k\lambda} + \frac{\partial}{\partial \varepsilon_{k\lambda}} \right), \quad \hat{\zeta}_{-k\lambda}^* = \frac{1}{\sqrt{2}} \left( \zeta_{k\lambda} - \frac{\partial}{\partial \varepsilon_{k\lambda}} \right).
\]

The function \( \Psi \) depends on all \( \zeta_{k\lambda} \), i.e., on the vector \( \zeta \). In the spirit of the theory of \( \Gamma \) operators \([10]\), we introduce the field operator \( \hat{\Phi}(\zeta, \varphi, t) \), satisfying the equation

\[
i \frac{\partial \hat{\Phi}}{\partial t} = \hat{P} + \hat{H}_\varphi \Phi - \frac{q}{m} \mathbf{p} \hat{A}_\varphi(\varphi) \Phi
\]

and the commutation condition

\[
[\hat{\Phi}(\zeta, \varphi, t), \hat{\Phi}^+(\zeta', \varphi', t') = \delta(\zeta - \zeta') \delta(\varphi - \varphi').
\]

We will say that this operator operates in the space \( \Gamma \), the transformation to which is accomplished by the operator \( O(t) = \Phi^+\phi_0 \). Here \( \phi_0 \) is the wave function of the electromagnetic in the space \( \Gamma \). The wave function of the system under consideration \( X = \varphi = \int \Phi^+ \Psi \phi_0 \), after transformation to the interaction representation, satisfies the equation

\[
i \frac{\partial \varphi}{\partial t} = -\frac{e}{m} \int \hat{\Phi}^+(\zeta, \varphi, t) \hat{\Phi}^+(\varphi, R, t) \hat{\rho}_\varphi A_\varphi(r) \exp(i\mathbf{p} \cdot \mathbf{r} - is\mathbf{p} \cdot \mathbf{r}) \frac{e^{-is\mathbf{p} \cdot \mathbf{r}}}{V^2}.
\]

We are interested in the matrix \( \rho = \langle \Phi^+ \Phi \rangle \) \([10]\). We introduce the matrix Green's function \([10, 11]\]

\[
\tilde{F}_{\mu\nu} = -i < T_{\zeta} \hat{\Phi}_\zeta(\zeta, \varphi, t) \hat{\Phi}_\zeta(\zeta', \varphi', t') \hat{S}_\zeta > \phi_0, l = 1, 2,
\]

\[
\hat{S}_\zeta = T_{\zeta} \exp \left[ i m \sum \Phi_{ij} \int \hat{\Phi}_\zeta^+ \hat{\Phi}_\zeta^+ \hat{\Phi}_\zeta \hat{\Phi}_\zeta \hat{A}_\zeta(\zeta') \hat{A}_\zeta(\zeta) \exp(i\mathbf{p} \cdot \mathbf{r} - is\mathbf{p} \cdot \mathbf{r}) d\mathbf{r} \right].
\]

By \( \phi_0 \) is meant here the state of the system before inclusion of the interaction of the field with the atoms of the medium. Expansion of \( \hat{S}_\zeta \) in a series and summation of the diagrams shows that \([10]\)

\[
\tilde{F}_{\mu\nu} = F_{\mu\nu} - i\rho_{\mu\nu}, \rho = \rho_{12} \text{ for } t = t'.
\]

All the terms containing the normal product of the operators \( \hat{\Phi} \) and \( \Phi^+ \) are assigned to \( \rho_{12} \); the remaining vacuum terms are contained in \( F_{\mu\nu} \). The statistical version \([12]\) of the Wick theorem is used for the operators \( \hat{\Phi} \) and \( \Phi^+ \). The functions \( F_{11} \), are found from the equations

\[
F_r = F_{11}, \quad F_{12} = 0, \quad F_{22} = F', \quad P_r = P_{11}, \quad P_{12} = 0, \quad P_{22} = P', \quad F_r = F' + F'P', \quad F_a = F', \quad P_a = P'.
\]