Bayesian Updating Revisited

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Bayesian updating methods provide an alternate philosophy to the characterization of the input variables of a stochastic mathematical model. Here, a priori values of statistical parameters are assumed on subjective grounds or by analysis of a data base from a geologically similar area. As measurements become available during site investigations, updated estimates of parameters characterizing spatial variability are generated. However, in solving the traditional updating equations, an updated covariance matrix may be generated that is not positive-definite, particularly when observed data errors are small. In addition, measurements may indicate that initial estimates of the statistical parameters are poor. The traditional procedure does not have a facility to revise the parameter estimates before the update is carried out. Alternatively, Bayesian updating can be viewed as a linear inverse problem that minimizes a weighted combination of solution simplicity and data misfit. Depending on the weight given to the a priori information, a different solution is generated. A Bayesian updating procedure for log-conductivity interpolation that uses a singular value decomposition (SVD) is presented. An efficient and stable algorithm is outlined that computes the updated log-conductivity field and the a posteriori covariance of the estimated values (estimation errors). In addition, an information density matrix is constructed that indicates how well predicted data match observations. Analysis of this matrix indicates the relative importance of the observed data. The SVD updating procedure is used to interpolate the log-conductivity fields of a series of hypothetical aquifers to demonstrate pitfalls and possibilities of the method.

KEY WORDS: Bayesian updating, geostatistics, kriging, linear inversion.

INTRODUCTION

Bayesian updating methods provide an alternate philosophy to kriging for characterization of input variables of a stochastic mathematical model. In this approach, a priori values of statistical parameters (for instance, mean and covariance) are assumed on subjective grounds or by analysis of a data base from a geologically similar area. As measurements become available during site investigations, "updated" estimates of these parameters are generated. The updated estimates can be used as input variables in further conditional simulations such as stochastic pore-pressure or contaminant transport modeling. Massmann and Freeze (1987) applied such a methodology in conditional simulations to...
quantify whether hydraulic conductivity measurements are cost-effective in reducing risk for owner/operators of landfill sites. Hachich and Vanmarcke (1983) applied Bayesian updating techniques in the interpolation of hydraulic head fields. Their algorithm first used an a priori hydraulic conductivity distribution to generate a mean hydraulic head field and covariance by a first-order second-moment method. Second, they updated this prior hydraulic head field with measurement values using the Bayesian procedure.

Such a technique would appear at first glance to offer considerable advantages over kriging in that no prior computation of variograms is necessary. However, a form of the spatial structure must be assumed. In a traditional implementation of the updating procedure (for example, Massmann and Freeze, 1987), an updated covariance matrix may possibly be nonpositive-definite due to finite machine precision and numerical roundoff. In addition, in cases where more is known about the form of the assumed covariances than their variances, a globally unique Bayesian update may not exist.

Neither of the above-mentioned studies performed sensitivity tests to determine how well the interpolated fields reproduced the true fields under a variety of a priori estimates of the parameters. In addition, a set of new measurements may indicate that the assumed random field is more variable than originally thought. Some mechanism for incorporating spatial information or the variance contained in new measurements into the original estimates is needed (Massmann and Freeze, 1988).

This paper presents a review of the Bayesian updating methodology and recasts the problem in a framework of generalized-linear inverse theory. A procedure based on a singular value decomposition (SVD) is presented. This method is efficient, stable, and accurate. In addition, an information density matrix can be formed from matrices associated with the SVD. The information matrix indicates how well the predicted data match observations. The SVD updating procedure is used to interpolate the log-conductivity field of a series of hypothetical aquifers to demonstrate the pitfalls and possibilities of the method.

BACKGROUND

The background to Bayesian updating rests with an application of Bayesian statistical inference (Bryson and Ho, 1969; Tarantola, 1987; Backus, 1988). Bayesian inference supposes that an observer can define a personal-prior probability distribution (pdf) of a model $m$. This pdf [denoted as $f_{pr}(m)$] can be defined on the basis of observations, personal experience or judgment. It can

3See Appendix A for notation.