To Be or Not to Be . . . Stationary? That Is the Question

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Stationarity in one form or another is an essential characteristic of the random function in the practice of geostatistics. Unfortunately it is a term that is both misunderstood and misused. While this presentation will not lay to rest all ambiguities or disagreements, it provides an overview and attempts to set a standard terminology so that all practitioners may communicate from a common basis. The importance of stationarity is reviewed and examples are given to illustrate the distinctions between the different forms of stationarity.

KEY WORDS: stationarity, second-order stationarity, variograms, generalized covariances, drift.

Frequent references to stationarity exist in the geostatistical literature, but all too often what is meant is not clear. This is exemplified in the recent paper by Philip and Watson (1986) in which they assert that stationarity is required for the application of geostatistics and further assert that it is manifestly inappropriate for problems in the earth sciences. They incorrectly interpreted stationarity, however, and based their conclusions on that incorrect definition. A similar confusion appears in Cliff and Ord (1981); in one instance, the authors seem to equate nonstationarity with the presence of a nugget effect that is assumed to be due to incorporation of white noise. Subsequently, they define "spatially stationary," which, in fact, is simply second-order stationarity.

Both the role and meaning of stationarity seem to be the source of some confusion. For that reason, the editor of this journal has asked that an article be written on the subject. We begin by noting that stationarity (at least as it is defined in the following section) refers to the random function and not to the data. It is this interchange that is the source of much of the confusion that appears in the literature. Moreover, different forms of stationarity, or perhaps we should say nonstationarity, are recognized. In some instances, authors have neglected to indicate which form they are using.

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STATIONARITY

In all of the following, assume that \( Z(x) \) is a random function defined in 1, 2, or 3 space (also referred to as a random field in some literature); \( x \) is a point in space, not just the first coordinate.

(Strong) Stationarity

\( Z(x) \) is stationary if, for any finite number \( n \) of points \( x_1, \ldots, x_n \) and any \( h \), the joint distribution of \( Z(x_1), \ldots, Z(x_n) \) is the same as the joint distribution of \( Z(x_1 + h), \ldots, Z(x_n + h) \) (Cox and Miller, 1965, p. 273–276).

This is not the form of stationarity that is usually referred to in the geostatistical literature, but it is essentially this form that is required for the application of nonlinear techniques such as disjunctive kriging and indicator and probability kriging. Note that this form of stationarity does not imply the existence of means, variances, or covariances. In nearly all instances, the data are represented as a (nonrandom) sample from one realization of the random function and hence can not be tested for stationarity. To refer to the stationarity (or nonstationarity) of the data implies a misunderstanding or a different definition from that given above. Lest we conclude that stationarity is too strong in all circumstances, most of statistics is based on at least a weak form of stationarity.

Second-Order Stationarity

Both because of the difficulty of testing for strong stationarity and the fact that it does not imply the existence of moments, a weaker form known as second-order stationarity often is used instead.

\( Z(x) \) is second-order stationary if \( \text{cov}[Z(x + h), Z(x)] \) exists and depends only on \( h \). This implies that \( \text{var}[Z(x)] \) exists and does not depend on \( x \); furthermore, \( E[Z(x)] \) exists and does not depend on \( X \) (Cox and Miller, 1965, p. 277, for details).

Obviously, stationarity does not imply second-order stationarity. Conversely, a stationary random function may not be second-order stationarity, as its first two moments may not be defined. Examples to illustrate this are given in Appendix A. Because stationarity is essentially untestable, as weak a form as possible is desirable to utilize and, as will be seen from a later example, even second-order stationarity may be strong. We turn to forms introduced by Math-ereron (1971, 1973).

Intrinsic Hypothesis

The form of stationarity implied by the intrinsic hypothesis is essentially second-order stationarity—not for the random function \( Z(x) \), but rather for the first-order difference, \( Z(x + h) - Z(x) \). Differences have been used in a number of places in statistics as well as in time series analysis (see, for example,