An exact solution is obtained for the relativistic collisionless kinetic equations describing a test plasma in the field of a strong plane gravitational wave. The gravitational wave induces in the plasma a longitudinal electric current whose amplitude is maximum at temperatures \( T_e \sim \frac{m_e c^2}{e} \). The interaction of gravitational waves with a system consisting of Boltzmann ions and degenerate electrons is also considered.

The interaction of gravitational waves (GW) with a medium is a sufficiently subtle and complex process, many aspects of which have not yet been studied. There is presently no physically clear model with which one could formulate the clear picture of the interaction of GW with a medium, which is particularly necessary for the theory of GW detection. Present calculations of GW detectors are based on phenomenological models of a medium in a gravitational field, which do not have a dynamic basis. This refers primarily to solid GW detectors since there is not yet a satisfactory theory of a solid body in relativistic gravitational fields.

It is important in this situation to analyze what happens to the medium with incidence of GW on the basis of some simple dynamic model of the medium. Plasma, the principles of the general relativistic theory of which have been formulated in works [1-4], is such a convenient object for investigating the processes of GW interaction with the medium. The response of a plasma to an external field is due to two factors — the interparticle interactions (Coulomb, nuclear, etc.) and the collective interactions taking place with the help of macroscopic fields. The latter factor is common to plasma and solids; the possibility of a rigorous dynamic description of plasma is due to the smallness of the correlation parameter (smallness of the average energy of the two-particle interaction as compared to the average kinetic energy of the particles).

The theory of the linear interaction of GW with plasma and gas has been formulated in [5-9]. The linear response of an isotropic homogeneous plasma to GW does not contain an electromagnetic component and is indistinguishable in this respect from the linear response of a gas of neutral particles. From the point of view of the theory of GW there is great interest in the problem of the nonlinear propagation of GW in plasma. This problem, however, is extremely complex, and we will consider in this article its simplified version, taking into account the effect of the GW on the plasma and neglecting the inverse effect on the GW of the gravitational field induced in the plasma. According to [5, 6] this approximation is valid for \( \tau \omega_g \ll 1 \), where \( \tau \) is the length of the GW packet or the period of the GW, \( \omega_g = \sqrt{\kappa e c^2} \), \( \kappa \) is the energy density of the plasma, and \( \epsilon = 8\pi G/c^4 \).

Under these conditions the phase velocity of the GW waves approaches the velocity of light, and any vacuum wave solution of the Einstein equations can be taken as the GW metric. Let us consider, e.g., a plane GW whose metric in an appropriate coordinate system can be reduced to the form

\[
ds^2 = (dx^1)^2 - (dx^1)^2 - A(u)(dx^2)^2 - B(u)(dx^3)^2 + 2C(u)dx^2dx^3,
\]

where \( u = x^4 - x^1 \) is the retarded time. For \( C = 0 \) Eq. (1) describes a plane-polarized GW with \( A(u) \) and \( B(u) \) related by the differential equation [10, 11]

\[
L'' = -2\beta L',
\]

where \( L = (AB)^{1/4} \), \( \beta = (1/4) \ln (A/B) \). The space described by (1) is flat for \( \beta' = 0 \), and the metric (1) before the arrival of the GW is pseudo-Euclidian:

\[
A(u) = B(u) = 1, \quad C(u) = 0, \quad (u < u_0).
\]
After passage of the GW pulse ($B' = 0$) there arises in the metric (3) at time $u \sim 1/B' c$ a coordinate singularity related to the degeneracy of the determinant of the metric tensor $g = AB - C^2$. For weak GW $A(u) = 1 + \xi(u); B(u) = 1 - \xi(u)$, where $\xi(u), C(u)$ are arbitrary functions of the retarded time.

1. Distribution Function of a Collisionless Plasma in a GW

If the frequency of the GW is much greater than the effective frequency of Coulomb collisions of particles in the plasma $v_{\text{eff}} \ll 1$, the effect of the GW on the plasma can be described with the Vlasov equations [3, 4, 12, 13]:

\[ p_i \partial_t f_a(x, p) + (e_a/c) F^i \partial p_i f_a = 0; \]
\[ F^i_{\mu, x} = -(4\pi/c) I^i; F_{(i \mu)} = 0, \]
where the vector of the current density $I^i$ is related to the distribution function $f_a(x, p)$ in the following manner:

\[ I^i(x) = \sum_a e_a c n_a \partial^i(x) = \sum_a e_a c \int p^i f_a(x, p) dP \]

Let $\xi_i, ..., \xi_3$ be the Killing vectors with motion along which the vector potential of the electromagnetic field $\varphi_i$ is conserved:

\[ L_{\varphi_i} = 0 \quad (r = 1, ..., s). \]

The integrals of the kinetic equation (4) are then [12, 14]

\[ C_r = (\xi^i, p); C_0 = \sqrt{(p, p)} = m_a c, \]

where $p^i = p^i + (e_a/c) \varphi_i$. The metric (1) allows three Killing vectors with whose help the following integrals of motion can be constructed:

\[ C_1 = p^i - p^i \equiv p_4 + p_1, \quad C_0 = \sqrt{(p, p)} = m_a c; \quad C_3 = -A (p^2 + (e_a/c) \varphi^2) + C (p^2 + (e_a/c) \varphi^2) + p_3 + e_a/c \varphi_3, \]

\[ C_3 = -B (p^2 + (e_a/c) \varphi^2) + C (p^2 + (e_a/c) \varphi^2) + p_3 + (e_a/c) \varphi_3, \]
where $q_i = (0, q_2(u), q_3(u), 0)$ is the potential of the transverse isotropic electromagnetic field automatically satisfying condition (7). An arbitrary function of these integrals is a solution of Eq. (4) if the nonlinear electric field induced in the plasma by the GW is not taken into account. We define this function so that before the appearance of the GW and electromagnetic wave in the plasma it transforms into the isotropic equilibrium solution. We thus obtain the exact distribution function describing the plasma in the field of strong gravitational and electromagnetic radiation propagating with the velocity of light:

\[ f_a(u, p) = (2\pi h)^{-3} \exp \left\{ -\frac{u}{c} T_a + c (2 T_a C_1)^{-1} (C_3 + C_i) \right. \]
\[ \left. + (C_1^2 + C_3^2) \right\} \pm 1^{-1}, \]

where $u = \text{const}$ is the chemical potential, "$+$" corresponds to fermions, and "$-$" to bosons.

2. Current Induced in the Plasma by the GW

Let us calculate the current density vector with the help of expression (6). It is convenient to transform to new variables in momentum space:

\[ \tilde{p}^i = C_1, \tilde{p}^2 = C_2; \tilde{p}^3 = C_3; D (p^i, p^2, p^3) \rightarrow D (\tilde{p}^i, \tilde{p}^2, \tilde{p}^3) = p^i |(AB - C^2)|^{-1}. \]

For a gas of Boltzmann particles, with consideration of the electrical neutrality of the plasma with $u < 0 (\sum e_a n_a = 0)$, we obtain: