width as a function of the required AOD frequency band was calculated from expression (10). With \( \lambda = 0.63 \text{ \mu m} \) and \( \Delta f = 500 \text{ MHz} \), it was 1.5 mm. To obtain a high electric field in the gap, the latter was chosen as small as possible and was limited by the size of the Airy spot of the light beam focused on the gap. With \( \lambda = 0.63 \text{ \mu m} \), the mean slit width was 30 to 50 \( \text{\mu m} \). With a power of 1 W and \( \lambda = 0.63 \text{ \mu m} \), the diffraction efficiency in the AOD developed was 10 to 30\% over the whole frequency range. The measured frequencies \( f_{1,2} \) shown in Fig. 4 are in good agreement with those calculated. The losses in the transformation of microwave into elastic-wave energy by the slit transducer described, computed from the value of the \( M_2 \) factor calculated earlier and the measured value of diffraction efficiency, was 10-15 dB.

LITERATURE CITED


ANALYTIC PROPERTIES OF THE SOLUTIONS OF TEUKOLSKY'S EQUATION

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A method is proposed for studying the low-frequency solutions of Teukolsky's equation, describing the behavior of massless fields in the vicinity of a Kerr black hole. Approximate basis solutions and an integral equation for exact solutions are constructed. The analytic properties of the exact solutions and the asymptotic behavior of the leading nonanalytic parts are investigated and the approximate solutions are estimated.

Teukolsky's equation [1] describes the behavior of massless fields in the vicinity of a rotating black hole, including the most interesting cases of a test electromagnetic field and perturbations of the gravitational field against the background of the Kerr metric. The analytic properties of the solutions of this equation as functions of frequency \( \omega \) determine, as is well known, the damping of the fields with time in the vicinity of a black hole. The vicinity of the point \( \omega = 0 \) is of greatest interest in this case. It is more difficult to study the case \( \omega \to 0 \) compared to the case \( \omega \neq 0 \) [2], since the approximate basis solutions, which are used for constructing the integral equation for the exact solutions, must give a quite good uniform approximation for \( \omega \to 0 \). In the present work, we construct such solutions by modifying the results that we obtained in [3] for the simpler case of a scalar field in the vicinity of a black hole without rotation. These solutions, although they are not written out explicitly, are expressed in terms of known functions by means of transformations with simple analytic properties and permit, in principle, obtaining all information concerning the exact solutions with \( \omega \to 0 \), including their explicit approximation as well.

We will use the representation of Teukolsky's equation in which the first derivative is omitted:

\[
d^2 \varphi/dr^2 + Q(r) \varphi = 0, \quad r \in (r_-, \infty),
\]

where

Here, $M$ and $a$ are, respectively, the mass and specific angular momentum of the black hole (it is assumed that $M > a$); the function $\varphi = \varphi_{\text{in}}(\omega, r)$ is related to the radial parts $\varphi_{R, n}(\omega, r)$ of the wave functions, arising after expansion with respect to spin spheroidal harmonics and Fourier transformation with respect to time, by the relation $\varphi = \Delta^{(s+1)}/2R$; the constants $\lambda_{s, n}$ are characteristic numbers in the equation for the spin spheroidal harmonics \cite{1, 2}; for our purposes, it is important that they are analytic with respect to $\omega$ in the vicinity of the point $\omega = 0$; $l$ determines the order of the multipole. The value of the spin $s$ corresponds for $s = 0$ to the case of a scalar field, for $s = 1$ to an electromagnetic field, and for $s = 2$ to perturbations of the gravitational field. In order to avoid complicating the presentation, we will limit ourselves to the case $s \geq 0$, since the solutions for positive and negative values of $s$ are related \cite{4} and, in addition, the case $s < 0$ is examined within the framework of the method proposed without any essential differences and leaves all the results of the present work unchanged.

In what follows, we will examine the approximate basis solutions on the segments $(r_+, r_0), [r_+, r_2], \text{ and } [r_2, \infty)$ (denoting them by I, II, and III, respectively), where the quantities $(r_+ - r_0)/M$ and $M/r_2$ are chosen to be sufficiently small.

1. Approximate Basis in the Vicinity of the Horizon

In the vicinity of the point $r = r_+(x = r - r_+)$

$$Q (r) = Ax^{-2} + B_+ x^{-1} + B_0 + B_1 x + ...$$

where

$$4A = [K_0 (M^2 - a^2)^{-1} - is]^{-1} + 1; K_+ = (r_+ + a^2) \omega - \alpha \Pi$$

and all coefficients are analytic for $\omega < 0$. It is easy to verify that from an equation of the form

$$\frac{d^2 \varphi}{dx^2} + \frac{A x^{-2} + C x^{\rho} + C_1 x^{\rho+1} + ...}{C_0 x^{\rho-1}} \varphi = 0$$

with the help of the substitution $\varphi (x) = \chi (y) \varphi (y)$

$$x = y \left(1 + a y^{m+2}\right), \chi (y) = (dx/dy)^1/2$$

it is possible to obtain with $4A - 1 + (m + 2)^2 \neq 0$ an equation of the form

$$d^2 \varphi \left(dy^2 + \frac{A y^{-2} + C y^{m+1} + C_1 y^{m+2} + ...}{C_0 y^{m-1}} \right) \varphi = 0,$$

assuming that

$$\alpha = -(2C m + 2) \left[4A - 1 + (m + 2)^2\right].$$

From an examination of the superposition of the transformations (3) and from (2), it follows that the substitution

$$x = y \left(1 + \sum_{\rho=1} a_\rho y^\rho\right), \chi (y) = \left(dx/dy\right)^{1/2}$$

with coefficients $a_\rho$ analytic for $\omega < 0$ (except for the separately examined case when $s \neq 0$ and $\alpha = 0$) permits constructing functions in region I

$$\varphi_i (x) = \chi (y) y^{i} (i = 1, 2); a_{1,2} = 1/2 \pm 1/2 \left[s + iK_+ \sqrt{M^2 - a^2}\right],$$

that satisfy the equation

$$d^2 \varphi/dx^2 + Q_1 (r) \varphi = 0,$$

where $Q_1 (r)$ is analytic for $\omega > 0$ and we have the following estimate uniform with respect to $\omega$:

$$|Q (r) - Q_1 (r)| \leq \text{const} x^{r-1}.$$