PARAMETRIC PHENOMENA IN SYNCHRONIZATION
OF THOMSON SYSTEMS. II

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It is shown that under conditions of a weak external impressed force the oscillation in the
locking range of a self-excited oscillator is compounded of self-oscillations and forced
oscillations with the same frequency but different phases, the self- and forced oscillations
acting one upon the other.

It was shown in Part I that for a weak resonance external impressed force the locking of a self-
excited oscillator described by the Van-der-Pol equation is caused by parametric excitation of self-oscila-
tions with the frequency of the impressed force (emf). The variable parameter of the system is the dif-
ferential resistance $R_I$ of the oscillator tube. The differential resistance $R_I$ is modulated by the forced
oscillations in the system.

In this case self-oscillations and forced oscillations with the same frequency but different phases
can exist simultaneously in the locking range. In the nonlinear parametric self-oscillating system under
study these oscillations will react one upon the other.

Let us assume to start with that within the locking range the forced oscillations can also be deter-
mined from Eq. (1.3)* (i.e., ignoring the effect on them of the self-oscillations). Equations (1.7) and (1.8)
are then also valid for the amplitude and phase of the self-oscillations (1.6) in the locking range.

Setting $dp/dr = d\theta/dr = 0$, we find from (1.7) and (1.8) the equation describing the family of ampli-
tude curves of the self-excited oscillator in the zone of parametric excitation of the oscillations. If the
amplitude of the impressed emf is small and $m^2 \ll 1$, the amplitude curves have the form of ovals, correct
to quantities of the second order of smallness:

$$
\rho^2 = 1 + \frac{5}{2} m^2 \pm 3 \sqrt{m^2 - 4 \xi^2},
$$

where $\xi = \sigma/e$. The width of the locking range and the real branch of the amplitude curves (1) are deter-
mined by the stability conditions. The curve bounding the region of stability [1]

$$
\rho^2 = 1 - 18 \xi^2 - 2m^2
$$

intersects the amplitude curve (1) at the points

$$
\rho^2 = 1 + \frac{5}{2} m^2,
$$

so that the condition

$$
\rho^2 = m^2 - 4 \xi^2 = 0
$$
determines the boundaries of the locking range. The upper branch of oval (1) proves to be stable.

When allowance is made in Eqs. (1.7) and (1.8) for quantities of the second order of smallness, we
obtain the following expression for the family of amplitude curves in the locking range:

*By (1.3) we mean Eq. (3) of Part I, and so on.
\[
\varphi^2 \simeq \left[ 1 + \left( \frac{3}{2} - 4m^2 \right) m^2 - (16 + 8m^2 - 48m^2) \xi^2 \right] \\
\pm \sqrt{9(m^2 - 4\xi^2) - 11m^4 + 368\xi^4 - 80m^2\xi^2}.
\] (3)

It follows from (3) that at the boundaries of the locking range, determined from the condition
\[
9(m^2 - 4\xi^2) - 11m^4 + 368\xi^4 - 80m^2\xi^2 = 0,
\] (4)
the amplitude of the excited oscillations is less than the amplitude \(a_0\) of the free-running oscillations.

From (2) and (4) the width of the locking range is given approximately by
\[
2\xi \simeq m,
\]
which corresponds to the known relationship between the locking band and the amplitude of the impressed emf: \(2|1 - \gamma| \equiv E/a_0\).

The solution of Eq. (4) for \(\xi^2\)
\[
\xi^2 \simeq 0.23m^2 \left( 1 - 2m^2 \right)
\]
shows that the relationship between the locking band and the amplitude of the impressed force is linear only at small values of \(m^2\). The locking band should increase only up to \(m^2 = 1/4\), after which it should decrease to zero at \(m^2 = 1/2\).

This result can be interpreted in the following manner. In the investigated nonlinear self-oscillating system subjected to an impressed force, the zone in which oscillations are parametrically excited degenerates with increasing amplitude of the forced oscillations into a "lobe" due to the increase in the damping in the system. Accordingly, for a resonance impressed force, just as in the case of a force at multiple frequencies, there is no "threshold" for the synchronizing external force but there is a "ceiling" [2]. In experiment, however, no reduction in the width of the locking range at fundamental frequency is observed with increasing amplitude of the impressed emf, let alone loss of synchronization. This lack of correspondence between calculation and experiment is connected with a change in the mechanism responsible for locking in the oscillator. In the case of small impressed emfs locking is caused by the parametric excitation in the system of self-oscillations with the frequency of the impressed force; in the case of large impressed emfs only forced oscillations remain in the system and the self-oscillations will be quenched. For a resonance force the self-oscillations are quenched at smaller impressed emfs than in the case of a force at multiple frequencies. Clearly, the value \(m^2 = 1/4\) determines the boundary between small and large impressed emfs.

The above calculations show how forced oscillations, determined in the linear approximation, influence the self-oscillations. A more complete picture of the behavior in the locking range can be obtained by allowing for the effect of self-oscillations on the forced oscillations. The solution of Eq. (1.1) taken in the form (1.2) we determine from the set of equations
\[
\dot{z} - z \left( 1 - x^2 \right) (1 - z^2) = 0, \quad (5)
\]
\[
\dot{y} - z (1 - x^2) \dot{y} + y = E \cos \gamma. \quad (6)
\]
It is not possible to solve this set in the general case. Utilizing the known fact that in the locking range of a self-excitied oscillator the only oscillations that exist are oscillations at the frequency of the impressed force, we investigate the solution of Eqs. (5) and (6) only in the locking range. We seek the solution of Eq. (5), which determines the self-oscillations in the system when subjected to forced oscillations \(y = B \cos (\gamma \tau + \varphi)\), in the form \(z = a \cos (\gamma \tau + \varphi)\); the solution of Eq. (6), which described forced oscillations when allowance is made for the effect on them of self-oscillations \(z = a \cos (\gamma \tau + \Theta)\), we seek in the form
\[
y = B \cos (\gamma \tau + \varphi).
\]

On solving Eqs. (5) and (6) by the asymptotic method of Bogolyubov and Mitropol'skii [1] we obtain two pairs of coupled truncated equations with which to determine the amplitudes \(a\) and \(B\) and the phases \(\Theta\) and \(\varphi\):
\[
\frac{da}{dz} = \frac{\varepsilon}{2} \left[ \left( a - \frac{a^3}{4} - \frac{aB^2}{2} \right) + \frac{aB^2}{2} \cos (\Theta - \varphi) + \frac{aB^2}{4} \cos 2(\Theta - \varphi) \right],
\]
\[
\frac{d\Theta}{dz} = \dot{\varphi} - \frac{aB^2}{4} \sin (\Theta - \varphi) - \frac{B^2}{8} \sin 2(\Theta - \varphi),
\]
\[
\frac{dB}{dz} = \frac{\varepsilon}{2} \left[ \left( B - \frac{B^3}{4} - \frac{a^2B}{2} \right) + \frac{a^2B}{4} \cos 2(\Theta - \varphi) - \frac{aB^2}{2} \cos (\Theta - \varphi) \right] - \frac{E}{2\gamma} \sin \varphi,
\] (7)