STATISTICAL SCREENING OF A LIGHT BEAM AND
FLUCTUATIONS IN THE TRANSPARENCY OF
DISPERSED MEDIA

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Results of mathematical statistics about the covering of a plane by circles are used for a quantitative description of transparency fluctuations. It is shown from a comparison of the theoretical and experimental results for the dispersion of fluctuations of passing radiation that statistical screening is the governing reason for the fluctuations of passing radiation in dispersed media. An empirical formula permitting the approximate computation of the dispersion with an accuracy sufficient for practical purposes is proposed for large optical depths.

1. The main peculiarity of the attenuation of a narrow light beam by a system of coarse scatterers is its quite definite fluctuational nature. Taking into account that the total fraction of scattered radiation recorded (compared to direct radiation) is comparatively small for narrow beams, the main effect specifying fluctuations in the passing signal should be considered the statistical screening of direct radiation by a random system of scatterers. This phenomenon is that for independent scattering, the attenuation by a randomly distributed system of scatterers in the beam path is equivalent to the attenuation by a system of overlapping screens in a plane layer with some effective transverse attenuation. The results of the mathematical statistics on the random tossing of circles on a plane [1] or on the model of granular structure in the photographic recording of an image [2] can be used for a quantitative description of the phenomenon. In such an approach the phenomenon of statistical screening of passing radiation takes into account both the fluctuations in the number of scatterers in the visible volume and the statistical change in their mutual spatial arrangement on the one hand, and multiple small-angle scattering of the radiation, on the other.

2. Let \( I(r_i) \) denote the intensity produced at a point with the radius-vector \( r_i \) by passing radiation of an optical beam in a plane receiving aperture. If the dimensions of this latter exceed the radius of correlation of the intensity fluctuations, then the fluctuations of the signal being recorded will be averaged. Let us introduce the function

\[
S(r) = \begin{cases} 
1 & \text{when } |r| < R, \\
0 & \text{when } |r| > R,
\end{cases}
\]

with the meaning of a pulsed reaction of the aperture to describe the averaging effect of a circular receiving aperture with radius \( R \). Then the signal recorded by the receiving system (the radiation flux) at the point \( r \) will be determined by [2-4]

\[
J(r) = \int_{-\infty}^{+\infty} I(r_i) S(r - r_i) \, dr_i.
\]

Correspondingly, we have for the fluctuating component of the signal \( J' = J - \langle J \rangle \)

\[
J'(r) = \int_{-\infty}^{+\infty} I'(r_i) S(r - r_i) \, dr_i,
\]

where \( I' = I - \langle I \rangle \), and for the root-mean-square of the fluctuations
The inner integral in (4) is evaluated easily and equals \([2-4]\)

\[
\left\{ \begin{array}{ll}
\frac{2}{\pi} \left( \arccos x - x \sqrt{1 - x^2} \right) & \text{for } x < 1, \\
0 & \text{for } x > 1.
\end{array} \right.
\]

To determine the function \(B_1(r)\) we take into account that the measurable optical diameter of scattering by particles is described by the expression \([5]\)

\[\sigma_{\text{meas}} = \sigma_0 \kappa(z, z_0),\]

for the recording of some fraction of forward-scattered together with the direct radiation, where \(\sigma_0\) is the scattering diameter computed by means of Mie theory formulas; \(z = \rho(z/L)\); \(z_0 = \rho\psi_1\); \(L\) is the geometric thickness of the scattering layer; \(\psi_1 \) is the viewing angle of the receiving system; and \(\rho\) is the Mie diameter. A system of scatterers with diameter \(\sigma_{\text{meas}}\) is equivalent to a system of circular overlapping grains of cross section \(\sigma_0\) (of radius \(a\)) and transparency \(h = 1 - k(z, z_0)\). The autocorrelation function \(B_1(r)\) for such a system with a Poisson distribution of the grain centers has been determined in \([6]\)

\[B_1(r) = \left\{ \begin{array}{ll}
0 & \text{for } r < a, \\
1 & \text{for } r > a.
\end{array} \right.
\]

Two asymptotic cases are easily obtained from (8). For the receiving aperture dimension \(R \gg a\) the function \(F(r/2a)\) in the integrand can be expanded in a series and we can limit ourselves to the first member. Then the variance is written as

\[\sigma_j^2 = \frac{8 - 2}{\pi R^2} \int_0^1 \left[ e^{\psi F(x)} - 1 \right] x dx, \quad R \gg a,
\]

which is analogous (with \(k = 1\)) to the expression obtained under the same assumptions in \([1]\).

In the other extreme case, when \(R \ll a\) (very narrow optical beams), the function \(F(r/2a)\) can be expanded into series. Limiting ourselves to the first member of the expansion, we obtain after integrating in (8)

\[\sigma_j^2 = \frac{8 - 2}{\pi R^2} \int_0^1 \left[ e^{\psi F(x)} - 1 \right] x dx, \quad R \ll a,
\]

which agrees with an analogous result obtained in \([2]\) for \(k = 1\).

3. Let us compare the results of computations by means of the formulas obtained above with existing experimental results for the fluctuations of passing radiation in dispersed media. Represented in Fig. 1 are the results of measuring the root-mean-square deviations of the signal (the square root of the normalized variance) as a function of the optical thickness of the scattering layer by means of the results in \([7]\) (the dots are for: \(\sigma_{\text{part}} = 2.2 \cdot 10^2 \mu\); \(k = 0.64\); \(R = 1 \text{ mm}\)) and in \([8]\) (crosses are for: \(\sigma_{\text{part}} = 2.8 \cdot 10^2 \mu\); \(k = 0.58\); \(R = 2 \text{ mm}\)). The solid curves 1 and 2 in Fig. 1 show the results of computations using the asymptotic formula (9), justified for the selected experimental conditions. It is seen from Fig. 1 that the computational data for the curve 1 agree with the experimental results in the whole range of measured variances. For curve 2 systematic discrepancies start with the optical thickness \(\tau = 16\). For large thicknesses such a discrepancy is naturally explained by the influence of multiple large-angle scattering, which is not taken into account in the computations.