The angular frequencies of oscillation of a particle in the plane of the orbit (17) and in the perpendicular direction (9) are measured by an observer aboard a drift-free satellite.

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LITERATURE CITED


NUMERICAL SOLUTION OF CLASSICAL EQUATIONS OF MOTION FOR A FIELD

IN THE INFRARED REGION

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A numerical study is carried out of equations of motion for the classical Yang-Mills fields described by a nonstandard Arbuzov-Alekseev Lagrangian.

In this communication, we discuss numerical solutions of the classical equations of motion of the gauge field for the group SU(2), corresponding to an effective gauge-invariant Lagrangian [1]:

\[ L_{\text{eff}} = \frac{1}{4 M^2} D^a b f^a b D^c F^c + \frac{\xi}{6 M^2} f^{abc} A^a F^b F^c. \]

(1)

It is claimed that this Lagrangian describes the infrared region of QCD and conforms with the behavior of the gluon propagator \( D(k) \sim M^2/(k^2)^2 \) (for \( k^2 \to 0 \)) [2]. This behavior may be a deciding factor in the solution of the confinement problem. Classical field theory for Lagrangians of type (1) has been studied in [3]. The equations of motion, corresponding to the effective Lagrangian (1), have the form:

\[ (2D^2 + \xi D^2 - [1 + \xi] D^2 D_0) A^a - g f^{abc} A^b A^c \]

(2)

where \( F^a = \partial_\mu A^a_\mu - \partial_\mu A^a_\nu + g f^{abc} A^b_\mu A^c_\nu \) is the field tensor and \( D^a_\mu = g f^{abc} A^b_\rho \partial_\nu + g f^{abc} A^b_\nu \partial_\rho \) is the covariant derivative. Using the Wu-Yang substitution for the components of the vector potential

\[ A^a_\mu = n^a f(r)/(rg), A^a_5 = \pm a(r). \]

(3)

and changing variables: \( r = (gM)^{-1} \exp \beta \) (\( gM \) is a dimensional parameter of the theory), we obtain the following equations of motion, expressed as equations for the functions \( f = y_1(s) \) and \( a = y_5(s) \):

\[ y'_1 = y_2; y'_2 = y_3; y'_3 = y_4; \]

\[ y'_4 = 6y_4 - (11 - 4y_1 y_2 - (12y_1^2 - 8y_1 y_5 - 6)y_2 + 4y_1 y_3 - 2y_1 [(1 - \xi) y_5 + (1 + \xi) y_3 + (5 + \xi)(2y_1 y_5 - y_3) + (1 + \xi) y_1] + e^{y_5}(s); y'_5 = y_6; y'_6 = y_7; y'_7 = y_8; \]

\[ y_1' = 6y_1 - 10y_1 + 3y_5 - 7y_5 - (2y_4 - 3y_5)(y_7 - y_6) + 3y_3y_5 - 6y_3y_6 + \]
\[ + 10y_3^2 - 3y_5^2 + 2y_3^2y_7 + 4y_3y_4(y_1 - y_3) - y_4[y_1' - 4 - y_1^2 + y_1[(y_1^2 - y_1^2)(1 - y_1^2) - y_5y_6^2 - 2y_5^2y_7 + y_5(4y_7 - 5y_5y_7 + y_7 - y_7)]) + J(s)e^{bs}. \]

Here, \( p(s) = (gM)^{-3}p(r), \) \( J(s) = (gM)^{-3}J(r), \) and the prime denotes a derivative with respect to the parameter \( s. \) With the parametrization (3), the field energy density \( T_{00} \) for the Lagrangian (1) has the form

\[ T_{00} = (gM)^4e^{-s}(3 - 0.5(y_3 - y_3)^2 - 3[y_1^2 + y_1^2 + (y_1 - y_1)(y_2 - y_2)] - \]
\[ - 2y_1^2y_1(y_5 - y_5) + (y_1 - y_1)(y_1 - y_1)(y_1 - y_1) + y_1^2 + y_1^2(1 + y_1^2) - \]
\[ - y_1^2(5 + 2y_1^2 + 3y_1) + y_1^2(1 + 3y_1^2 - 4y_1) + 6y_1y_2 + (y_2 - y_2)^2 + \]
\[ + 8y_3y_6(1 - y_5^2 + 2y_5^2) + 2y_3y_6(y_3 - y_6) - 6y_3^2y_7(y_7 - y_7) + \]
\[ + \xi[(y_1 - 1)(y_1 + y_1y_1) + 2y_1y_1y_1(y_1 - y_1)] \].

It has been shown in [4] that there exist solutions of the nonlinear system of equations (4) that can be represented as series with respect to \( R^{-1} \), where \( R = r_{gM}. \) Then, the expansion with respect to inverse powers has the following form:

\[ a(R) = 1 + \sum_{n=1}^{\infty} a_n/R^n, \quad f(R) = \sum_{n=1}^{\infty} f_n/R^n. \]

We have performed a numerical analysis of solutions of the system (4), using the form of solutions (6). We have assumed that for large values of the dimensionless variable \( R, \) i.e., in the asymptotic region (in reality, a large value of \( R \) is \( R = 10 \)), the first terms of the expansion (6) give almost complete information about the solution of the type considered (naturally, with a certain accuracy). Choosing the coefficients of the first terms of the expansion [4] in accordance with the homogeneous equations of motion for the field:

\[
\begin{align*}
\alpha_1 &= \alpha; \\
\alpha_2 &= \beta; \\
\beta_1 &= \beta; \\
\beta_2 &= \beta (37 + i)/36
\end{align*}
\]

and taking initially \( R = 10, \) we have sought a solution of the homogeneous system (4), continuing until the value \( R = 0.5. \) The parameters \( \xi, \alpha, \) and \( \beta \) were chosen in the following way: \( \xi = -0.5, -1, +1; \) \( \alpha \) ranged from -6 to 6 in steps of 0.5; \( \beta \) was varied with the same step size but within the range from 0 to 6. We shall show that the accuracy of prescribing the initial conditions with the help of the formulas (7) decreases for large values of the parameters \( \alpha \) and \( \beta. \) For example, for \( R = 10, \) the percentage accuracy for the first four functions \( y_1 \) is in the order of \( \alpha^2 \) or \( \beta^2. \) Clearly, by using the recurrence relations obtained earlier for the coefficients \( a_n \) and \( f_n, \) one can improve the accuracy of the calculation to any desired level by taking as many terms of the expansion as needed. We note that an interesting exact relation \( f(-\beta) = -f(\beta) \) follows from (4) if no sources are present; there is no analogous relation for the parameter \( \alpha. \) The results of the calculations for the functions \( a \) and \( f, \) (for \( \xi = 1, \) \( \alpha = \beta = 1 \)) are shown graphically in Fig. 1. Changing the values of the parameters \( \xi, \alpha, \) and \( \beta \) does not qualitatively change the behavior of the functions \( a \) and \( f; \) its only consequence is a change in the amplitude of the maximum of the function \( a. \) Thus, for example, an increase in value of the parameter \( \beta \) decreases the magnitude of the maximum of \( a \) and shifts it toward the origin of coordinates. It has been noted that if, initially, one takes \( f = 0 \) and \( a = 1 + 6a_n, \) then the system goes arbitrarily rapidly from that state (strictly speaking, from that solution) to a different one. At the same time, if the function \( a \) is chosen initially equal to one, then the system remains in that state (solution). This result is obtained quite simply if one considers a corresponding linearized system of equations near the solution \( f = 0 \) and \( a = 1. \) This property of the solution for the equations of motion may lead to nontrivial conclusions concerning the physical properties of the system described by the corresponding effective Lagrangian. One of these conclusions is the instability of a purely chromomagnetic field configuration. Including fluctuations for the \( a \) component leads to the appearance of the chromoelectric...