The role of diffusion of the components of the gas mixture is the generation mechanism of a waveguide CO\(_2\) laser is analyzed. A method is proposed for reducing the multicomponent problem of linear diffusion with dissipation to a one-component problem. The superposition of relaxation modes for a cylindrical waveguide is considered. The diffusion contribution to the loss of electromagnetic energy is estimated for the case of the field hybrid mode EH\(_{11}\) of a dielectric waveguide. The rapid increase in the loss for a SiO\(_2\) waveguide tube and the observation of the filtration properties of the laser in conditions when the diffusion is taken into account are noted.

1. Recently the steady attention of researchers has been attracted by CO\(_2\)-gas waveguide lasers operating either in a slow pumping regime, or else in conditions of static saturation [1-6]. It is well known that the intensive development of this line of research was stimulated by the prospects of creating small-scale autonomic lasers with a bandwidth of up to tens of gigahertz and power of up to tens of watts with a wide returning range, self-synchronization of the modes and a simple realization of a stable one-frequency regime. Already both cw and pulse generation (pulse repetition frequency several kHz) has been obtained. The waveguide principle has also been extended successfully to gas mixtures whose active medium is composed of molecules of CO [7], CH\(_3\)OH, CH\(_3\)F [8], and thus already includes a whole set of spectral intervals in the region 5-200 \(\mu\)m.

Due to their special constructive features, waveguide lasers have very specific kinetics, which have not yet been studied sufficiently. The simple similarity laws [9] often cannot be used even for rough estimates of the laser parameters [5]. Effects may take a prominent place which, for lasers of more traditional type, it is of no great importance or even quite superfluous to take into account. Even writing down the initial equations of oscillatory relaxation in the system of multiatomic molecules under consideration becomes far from trivial.

This paper considers certain questions of the theory of a CO\(_2\) laser associated with the necessity of correctly taking into account diffusion in the thin cooled capillary channel which serves both to confine the plasma of the discharge and as a waveguide for coherent radiation with wavelength \(\lambda = 10.6\ \mu\)m. Christensen et al. [10] attempted to find means of increasing the saturation flux \(I_0\) as a result of diffusion of the molecules into the active volume, and then to estimate the power taken from unit length of the column, using the expression

\[ W/L = \beta r_0^2 \frac{\beta}{\epsilon} \]

Here \(\beta\) is the unsaturated gain and \(r_0\) is the radius at the intensity level \(\epsilon^{-1}\). However, Christensen et al. [10] start from over-simplified assumptions regarding the diffusion mechanism, taking into account neither the superposition of relaxation modes nor the multicomponent nature of the problem, nor the special form of the formulation of the boundary conditions.

2. We shall first indicate a simple means of reducing the multicomponent problem of linear diffusion with dissipation, which has already been considered partially in [11].

If $t_D$ is the characteristic path of the diffusion length, estimated for an excited nitrogen molecule (electrode state $1^1\Sigma^+$, oscillatory level $\nu = 1$) moving towards the wall of the waveguide, then for the differences of absolute values of the fundamental dissipative coefficients $\alpha_i$, $\alpha_k$, we can write $|\alpha_i - \alpha_k| < t_D^{-1}$. We thus have the system of equations [11]

$$\frac{\partial n}{\partial t} = \sum_{k=1}^{p} D_{ik} \nabla^2 n_k - \alpha_i n_i,$$

(2)

where $D_{ik}$ are the diffusion coefficients which can to a close approximation be treated as scalar quantities connected with the corresponding Onsanger relationships [12].

The transformation of Eq. (2) to a system of $p$ independent equations is effected using a procedure of diagonalization of matrix $D$ which is analogous to that which is widely used, for example, in solving problems of quantum mechanics. Let us consider some nonsingular matrix $g$, whose rank coincides with that of $D$. Multiplying the equations of the system (2) by elements $g_{ik}$, summing with respect to $i$ and replacing the suffix $i$ by $k$ in terms which do not contain double sums, we find

$$\frac{\partial}{\partial t} \sum_{k=1}^{p} g_{ik} n_k = \text{div} g_{ik} \nabla n_k - \alpha_i n_i,$$

(3)

In more compact form, we obtain

$$\frac{\partial G_j}{\partial t} = H_j \nabla^2 G_j - \alpha G_j, \quad j = 1, 2, \ldots, p,$$

(4)

where

$$G_j = \sum_{k=1}^{p} g_{kj} n_k,$$

$$H_j = \frac{1}{\sum_{i=1}^{p} g_{ij} D_{ik}}.$$

(5)

In (4) there are now no "linking" terms. The elements of the diagonalized matrix $H$ are found from the obvious condition $\det (D_{ik} - H \delta_{ik}) = 0$, and the solution of (2) takes the form

$$n_i = \sum_{j=1}^{p} g_{ij}^{-1} G_j,$$

(6)

($g^{-1}$ is an inverse matrix). The problem then reduces to the solution of the equation

$$\frac{\partial n}{\partial t} = D n + \alpha n,$$

(7)

3. For a cylindrical waveguide ($r$ and $z$ are the radial and axial coordinates) with an axially symmetric distribution of the particles, the solution of Eq. (7) takes the form

$$n(r, zt) = \sum_{\mu} \sum_{\nu=1}^{\infty} A_{\mu\nu} J_0 (\mu \nu r) \cos (\nu \pi z) \exp (-t \gamma_{\mu\nu}),$$

(8)

where $\mu$, $\nu$, and $\gamma_{\mu\nu}$ may be found from the expressions

$$J_0 (\mu \nu R) = 0,$$

$$\nu = (2n - 1) \pi L^{-1},$$

$$\gamma_{\mu\nu} = \{D (\mu^2 + \nu^2) + z \}^{-1}.$$

(9)

$L$ and $R$ are the length and radius of the waveguide; $J$ is the Bessel function symbol.

Turning to the parameters of the devices described in the works cited above, it is not difficult to show that the rate of decrease of the matrix elements $\gamma_{\mu\nu}$ as $m$ and $n$ increase is, as a rule, insufficient to lie within the bounds of the traditional approximation $\gamma_{\mu\nu} = \tau_{11}$, thus ignoring the superposition of relaxation modes. It is relevant to note that a similar situation arises in the problem on longitudinal relaxation