EFFECT OF A LONGITUDINAL WAVE ON THE
SELF-RESONANT MOTION OF ELECTRONS

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The effect of the longitudinal component of an electromagnetic wave on the self-resonant motion of electrons in waveguides is considered. The equation of motion for electrons in a plane, longitudinal-transverse, circularly polarized wave has been solved by the method of successive approximations with allowance for the longitudinal integral of motion present in such a wave. The expansion parameter is the ratio of the amplitudes of the longitudinal and transverse components of the wave. For the case \( n < 1 \) the longitudinal field of the wave does not change the nature of the electron motion, but only induces weak oscillations about a solution that does not include the longitudinal field. For \( n > 1 \) with spontaneous phasing of the electrons a criterion is obtained for neglecting the effect of the longitudinal field, and numerical estimates are given.

Electromagnetic waves were generated in [1] by induced bremsstrahlung of relativistic electrons under self-resonant motion [2, 3]. The corresponding self-consistent problem was solved in [4] for a purely transverse wave, while in [5, 6] some methods for maintaining self-resonant motion in waveguides were investigated, but the longitudinal wave component was also not taken into account. However, this component can lead to the breakdown of resonance and thus change the nature of the electron motion considerably. This problem is the subject of the present paper.

We will show, first of all, that in a plane wave described by the potential

\[
A = A(\theta); \quad \varphi = \varphi(\theta); \quad \Theta = t - nz; \quad n = \text{const},
\]

there are three exact integrals of motion for a charged particle. Two of them are projections of the generalized transverse momentum of the particle, which is conserved due to the cyclic nature of the transverse coordinates \( x, y \)

\[
P_\perp + A_\perp = C = \text{const},
\]

where the sign \( \perp \) denotes the transverse components of the vectors, the particle momentum \( p \) is divided by \( mc \), and the potential is expressed in units of \( mc^2/e \). Furthermore, all the velocities and lengths are divided by \( c \).

The third integral of motion is less trivial. It has been obtained in [7] for purely transverse waves. In the presence of the longitudinal components \( A_z \) and \( \varphi \) the corresponding integral of motion follows from the equation for the change in energy

\[
\frac{d\varphi}{dt} = -n \frac{\partial A_z}{\partial t} - u_z \frac{\partial \varphi}{\partial z}
\]

and the longitudinal projection of the equation of motion

\[
\frac{dp_z}{dt} = \frac{\partial A_z}{\partial t} - \frac{\partial \varphi}{\partial z} + u_z \frac{\partial A_\perp}{\partial z}
\]

with allowance for

\[
\frac{d\Theta}{dt} = 1 - nu_z; \quad \frac{\partial}{\partial t} = \frac{d}{d\Theta}; \quad \frac{\partial}{\partial z} = -n \frac{d}{d\Theta},
\]
and is written

\[ Y = n (\gamma + \varphi) - p_x - A_z = \text{const.} \]

In the presence of a longitudinal constant magnetic field the integral of motion \( C \) vanishes, and the integral of
motion \( Y \) remains in force since the longitudinal magnetic field is not present in the equations from which it is
derived.

Generally, the longitudinal field of the wave is inhomogeneous over the transverse cross sections in
waveguides, but for simplicity we will neglect this inhomogeneity on the basis that with sufficiently small Larmor
radii and with the radial drift neglected the entire particle trajectory lies in a region where the longitudi-
nal field can be considered to be independent of the transverse coordinates.

The effects associated with the self-resonant interaction appear most clearly in circularly polarized
waves, which is the reason they were studied in the works cited above. Therefore, we will consider an elec-
 tromagnetic wave which is described by the potential

\[ A_x = \gamma \cos \varphi; A_y = -\lambda \sin \varphi; \Phi = A_z - n \varphi = \beta \sin \varphi; \]

\[ \lambda = eE_\perp / m c \omega; \beta = eE_z / m c \omega; \varphi = \omega \Theta, \]

the transverse component is circularly polarized, and the longitudinal component varies harmonically in time.
In (1) the terms \( E_\perp \) and \( E_z \) are the amplitudes of the respective fields.

If the wave (1) propagates along a magnetostatic field \( B \), then the electron equation of motion in the pro-
jections becomes

\[ \frac{dp_x}{d\tau} = -\frac{dA_x}{d\tau} \quad \frac{dp_y}{d\tau} = -\frac{dA_y}{d\tau} \quad \frac{dp_z}{d\tau} = -\frac{1}{\gamma - np_z} \left[ np \frac{dA_\perp}{d\tau} + \gamma \frac{d}{d\tau} \Phi \right], \]

where \( \Omega = eB/(mc\omega) \). These equations will be solved by the method of successive approximations in a small
parameter, which is the ratio of the amplitudes of the longitudinal and transverse fields \( \delta/\lambda \ll 1 \).

As the zero-order approximation, we take the motion considered in [1, 5, 6], in which self-resonant
amplification of the wave occurs. We assume that the resonance condition is satisfied for the zero-order
approximation:

\[ \Omega = \gamma_0 - np_{z0}, \]

and this is obtained by varying a constant magnetic field in the longitudinal direction [5]. If condition (2) is
satisfied, then the zero-order solution is written

\[ p_{x0} := \gamma \sin \varphi + p_{\perp0} \cos (\varphi - \varphi_0); \quad p_{y0} = \gamma \cos \varphi - p_{\perp0} \sin (\varphi - \varphi_0); \]

\[ \Omega_0 = \gamma_0 \frac{1}{1 - n^2} + \frac{1}{1 - n^2} \left[ \frac{1}{4} \frac{1}{1 - n^2} - (\gamma_0^2 - \gamma_0^2) \right] \]

\[ p_{\perp0}^2 = \gamma^2 + 2p_{\perp0} \gamma \sin \varphi; \quad Y_0 = n_{\perp0} - p_{\perp0}; \quad p_{x0} = n (\Omega - 1)(1 - n^2). \]

The linear approximation over the longitudinal field is described by the equations

\[ \frac{dp_{x1}}{d\tau} = \frac{p_{x1} (1 - n^2) + \Phi}{\Omega n} \quad \frac{dp_{y1}}{d\tau} = -\frac{p_{y1} + p_{y0} (1 - n^2) + \Phi}{\Omega n} \]

\[ \frac{dp_{z1}}{d\tau} = \frac{n p_{z0}}{\Omega} \frac{dA_\perp}{d\tau} \quad Y_0 + p_{x0} \frac{d}{d\tau} + p_{x0} (1 - n^2) + \Phi \]

\[ p_{z0} \frac{dA_\perp}{d\tau} \]

It follows from Eq. (5) that

\[ \frac{dp_{x1}}{d\tau} - p_{x0} \frac{dp_{y1}}{d\tau} = p_{x0} p_{z0} - p_{x0} p_{z0}. \]