RELATIVISTIC CORRECTION TO SINGLE-PARTICLE NEUTRON LEVELS IN A HARMONIC OSCILLATOR WELL

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Relativistic corrections to mass and potential energy are calculated in the first approximation of perturbation theory for single-particle levels in a harmonic oscillator well. On the average, these corrections are not large, but increase greatly with increase in the main and orbital quantum numbers. For the ls state the relativistic correction is of the order of 0.01 MeV, while for 3p, we have 0.4 MeV. Thus, for some states the relativistic correction approaches the value of the spin-orbital interaction and must be considered in calculating single-particle energy levels.

INTRODUCTION

The nucleus is a nonrelativistic system. This assertion is based on the fact that the nucleon binding energy is much less than its rest mass, and has been confirmed by application of nonrelativistic theory with use of phenomenological potentials.

However, as was shown in [1], the intense repulsion between nucleons and the large value of the spin-orbital coupling indicate the necessity of considering relativistic effects.

In the present study we will consider the relativistic correction to neutron mass and potential energy to determine how applicable the nonrelativistic approximation is to calculation of single-particle energy levels. For simplicity, we have calculated the corrections for the single-particle neutron levels of a harmonic oscillator well

\[ V(r) = \frac{m_0^2 r^2}{2} - V_0, \]  

(1)

since in this case the radial wave function is known.

SEMRELATIVISTIC HAMILTONIAN

According to [2] the semirelativistic Hamiltonian has the form:

\[ \hat{H} = \frac{\hat{p}^4}{2M} - \frac{\hat{p}^1}{8m'c^2} \frac{dV(r)}{dr} \frac{d}{dr} + V(r) + \frac{1}{2m'c^2} \frac{v}{r} \frac{dV(r)}{dr} (sl). \]  

(2)

If we eliminate the second and third terms on the right

\[ \hat{H}_p = -\frac{\hat{p}^1}{8m'c^2} - \frac{\hbar^2}{4m'c^2} \frac{dV(r)}{dr} \frac{d}{dr}, \]  

(3)

which consider the relativistic correction to the mass and potential energy respectively, we are left with the Schrödinger equation with spin-orbital interaction.

Due to relativistic effects the spin-orbital interaction intensifies in the potential well

\[ V_{sl}(r) = \frac{\lambda}{2m'c^2} \frac{1}{r} \frac{dV(r)}{dr} (sl), \]  

(4)

where \( \lambda \) is usually selected from experimental data.

We will now reduce \( \hat{H}_{\rho} \) to a concrete form. To do this, we express \( \hat{p}^4 \) in the following manner:

\[
\hat{p}^4 = 2m \hat{T} \cdot 2m \hat{T},
\]

where the kinetic energy operator is expressed in the standard manner:

\[
\hat{T} = -\frac{\hbar^2}{2m} \frac{1}{r^2} \frac{d}{dr} \left( r^2 \frac{d}{dr} \right) + \frac{\hat{L}^2}{2mr^2}.
\]

Substituting Eq. (6) in Eq. (5), after simple transformations we obtain

\[
\hat{p}^4 = \hbar^4 \left[ \frac{d^4}{dr^4} + \frac{4}{r} \frac{d^3}{dr^3} - \frac{\hat{L}^2}{\hbar^2} \left( \frac{2}{r^2} \frac{d^3}{dr^3} + \frac{2}{r^4} \right) + \frac{\hat{L}^4}{\hbar^4} \frac{1}{r^4} \right].
\]

From this and Eq. (3) we obtain the relativistic energy correction operator

\[
\hat{H}_\rho = -\frac{\hbar^4}{8m^2c^2} \left[ \frac{d^4}{dr^4} + \frac{4}{r} \frac{d^3}{dr^3} - \frac{\hat{L}^2}{\hbar^2} \left( \frac{2}{r^2} \frac{d^3}{dr^3} + \frac{2}{r^4} \right) + \frac{\hat{L}^4}{\hbar^4} \frac{1}{r^4} \right] - \frac{\hbar^2}{4m^2c^2} \frac{dV(r)}{dr} \frac{d}{dr}.
\]

We will calculate the relativistic correction for single-particle energy levels in the first approximation of perturbation theory

\[
\Delta E_\rho = \int_0^{\infty} \tau_{nl}(r) \hat{H}_\rho \tau_{nl}(r) r^2 dr,
\]

using radial wave functions \( \tau_{nl}(r) \) [3] for a spherically symmetric harmonic oscillator.

With consideration of spin–orbital interaction the energy states are defined by the well-known expression [4]:

\[
E_{Nlj} = -V_0 + \left( N + \frac{3}{2} \right) \hbar \omega - a_{lj} \lambda \omega^2;
\]

\[
a_{lj} = \frac{\lambda}{2} \left( \frac{\hbar}{mc} \right)^2 \begin{cases} l, j_1 = l + \frac{1}{2} \\ - (l + 1), j_2 = l - \frac{1}{2} \end{cases};
\]

\[
N = 2n + l - 2.
\]

It is evident here that the relativistic correction \( \Delta E_\rho \) for \( n^2j_1 \) and \( n^2j_2 \) will be identical since the state \( n^2 \) corresponds to a relativistic correction \( \Delta E_\rho \) of specified value, Eq. (9).

**NUMERICAL RESULTS AND CONCLUSIONS**

The quantities \( V_0 \) and \( \omega \) were calculated by matching the energy, Eq. (10), to the 2s and 3s single-particle levels of the nucleus \( ^{197}\text{Au} \) [5]:

\[
\omega = 1.0903 \cdot 10^9 \text{ rad sec}^{-1};
\]

\[
V_0 = 56.327 \text{ MeV},
\]

with the spin–orbital interaction constant \( \lambda = 16.53 \) being selected from the difference between the \( l_{3/2} \) and \( l_{1/2} \) energy levels. Because of this parameter selection the energies of some single-particle levels coincide with the neutron levels of the \( ^{197}\text{Au} \) nucleus given in [5] although that study used the Saxon–Woods potential. The goal of the present study is