Maximum Entropy and Shear Strain of Shear Zone

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The principle of maximum entropy can be used to determine the shear strain in natural shear zones. When the margin of a shear zone is assumed, the principle leads to the truncated exponential distribution of the shear strain. If \( x \) is the distance remote from the shear zone center, which possesses the maximum shear strain, the shear strain \( \gamma(x) \) is given by

\[
\gamma(x) = \gamma_0 \frac{e^{-\beta x} - e^{-\beta_0}}{1 - e^{-\beta_0}}
\]

where \( \gamma_0 \) is the maximum shear strain and \( \chi_0 \) is the boundary distance. This relationship agrees with the observed data remarkably well. Further given no margin to distance, this relation generates the Becker's relation \( \gamma(x) = \gamma_0 e^{\beta x} \) under the condition \( \beta > 0 \). This truncated exponential distribution function which fits the observed data remarkably well is expected to be valid for the strain analysis of natural shear zones.

KEY WORDS: shear strain, maximum entropy principle, truncated exponential distribution.

INTRODUCTION

In rocks deformed by natural tectonic processes, it is usual to find that the finite strain state varies from locality to locality. In some deformed rocks high strain states are localized within approximately planar zones commonly known as shear zones (Ramsay and Graham, 1970). There is the variation in orientation of the long axis of strained objects in shear zones and this geometry of shear zones shows a foliation trajectory which is an approximate sigmoidal form (Ramsay and Graham, 1970; Ramsay and Huber, 1983; Dennis and Secor, 1987, 1990; Dennis et al., 1987; Jun-Yuan, 1987). Most geometrical studies of shear zones have been concentrated on estimating the strain state over deformed regions. This technique of strain analysis has led to great advances in our understanding of the deformation within tectonic structures and is likely to continue to do so. Moreover, the knowledge of the strain-distribution within shear zones must lead to the mechanics of formation of the tectonic structure. Although the various
techniques on strain analysis of deformed rocks have been studied by many researchers, there are few studies on the function of the strain profiles across shear zones. Formerly, Becker (1882) described the shear strain within shear zones by Becker's relation
\[ \gamma(x) = \gamma_0 m^{-x} \] (1)
where \( \gamma(x) \) denotes the shear strain at the distance \( x \) remote from the maximum shear strain position (shear zone center), and \( \gamma_0 \) is the maximum shear strain and \( m \) is constant (see Appendix). Then Jun-Yuan (1987) presented the similar relation given by
\[ \gamma(x) = \gamma_0 e^{-\varphi|x|^\omega} \] (2)
where \( \varphi \) and \( \omega \) are constants. But Becker's relation and Jun-Yuan's relation lie in inaccuracy near the margin of shear zones and have few theoretical bases. These are the major drawbacks to Becker's relation and Jun-Yuan's relation and problems one should not ignore.

The principle of maximum entropy provides a formal procedure for generating probability distributions from incomplete information. This principle is originated by Shannon (1948a, b). Further, many discussions and applications may be found in fields such as traffic engineering, mechanics of granular media, hydrology and quantum mechanics, and so forth (Jaynes, 1957a, b; Brillouin, 1956; Good, 1963; Katz, 1967; Tribus, 1969; Osteyee and Good, 1974; Ulrych and Bishop, 1975; Collins and Wragg, 1977; Brown, 1978; Skilling and Gull, 1985). These studies indicate that the usefulness of the principle follows the improvement of the research technique. Therefore, this paper will mention briefly the principle of maximum entropy and show the application of it in the determination of strain profiles across natural shear zones.

**PRINCIPLE OF MAXIMUM ENTROPY**

The principle of maximum entropy can be formalized by Shannon (1948a, b) and Jaynes (1957a, b) as follows:

Let \( n_i \) be a discrete random variable and let \( p_i = p_i \ln p_i \) be the associated probability distribution. Then, the entropy \( S \) is defined as
\[ S = - \sum_{i=0}^{n} p_i \ln p_i \] (3)
where \( \ln \) signifies the natural logarithm. The entropy \( S \) is a measure of the information content for a discrete set of probabilities \( \{p_i\} \) and let the distribution \( p_i \) be subject to the following constraints:
\[ \sum_{i=0}^{n} p_i = 1, \quad 0 \leq p_i \leq 1 \] (4)