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ON THE VALUE OF THE METRIC TENSOR AND THE ENERGY-MOMENTUM TENSOR
COMPONENTS ON THE BOUNDARY OF THE R AND T DOMAINS

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The case of the presence of matter in the R domain near the R-T boundary is ex-
amined. Using the relations between fixed and nonsingular reference systems at
the boundary, estimates are obtained for the behavior of the velocities and the
metric tensor components in the fixed reference system on the boundary of the R
and T domains. On this basis estimates are obtained for the behavior of the ener-
gy-momentum tensor components on the R and T boundary in the fixed reference sys-
tem and the locally Lorentz reference system.

The problem of the existence of black holes attracts constant attention at this time.
In connection with the final stage of development of gravitational collapse, an interesting
question arises: What if the collapse had been internal from the very beginning (see [8])?
That is, what if the formation of a kind of black hole appears within a spherically symmetric
object at the final stage of its development (in contrast to the consideration in [8], its
beginning can be sufficiently slow with the withdrawal of the initially collapsing sphere
from the appropriate gravitational radius)? (In the particular case when the thickness of
the external layer is zero we would arrive at the usual black hole.) What characteristics
should the surface of the internal black hole possess? How will the substance on such a sur-
face behave? To some extent the results of this paper answer these questions. Naturally,
many interesting questions raised by the results still remain, for instance, what will occur
with the substance external relative to such a surface from the viewpoint of a remote and
proper observer, what are the conditions preceding the beginning of internal collapse, etc.
We perform the analysis in the language of R and T domains [2]-[4], and in this sense the re-
results of the paper are of greater generality, and emerge outside the framework of questions
associated with the problem of the final stage of internal gravitational collapse.

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1. Let us consider a spherically symmetric distribution of matter. In this case the metric can be written in the form

\[ ds^2 = g_{00}(dx^0)^2 + g_{11}(dx^1)^2 - (dx^0)^2 \left[ (dx^2)^2 + \sin^2 x^3 (dx^3)^2 \right], \] (1)

where \( g_{00} = g_{00}(x^0, x^1) > 0 \), \( g_{11} = g_{11}(x^0, x^1) < 0 \), and \( x^0 \) is a time coordinate but \( x^1, x^2, x^3 \) are space coordinates. Let us designate as fixed the reference system (RS) associated with the metric (1).

Writing the Einstein equations for the metric (1) (see [1]) and integrating them with initial conditions on the boundary of the substance distribution where the external Schwartzchild solution holds, we obtain

\[ \mathfrak{g}_{11} = - \left( 1 - \frac{r_s}{x^1} \right) + \frac{a}{x^1} \int_{x_0^1}^{x_0^1} \frac{T_0^0 (x^1)^2 dx^1}{x_0^1} - \frac{a}{x^1} \int_{x_0^1}^{x_0^1} \frac{T_0^0 (x^1)^2 dx^1}{x_0^1}, \quad a = \frac{8\pi G}{c^4}, \] (2)

and

\[ g_{00}\mathfrak{g}_{11} = - \exp \left[ a \left( \int_{x_0^1}^{x_0^1} T_0^0 x^0 dx^0 - \int_{x_0^1}^{x_0^1} T_0^0 x^0 dx^0 \right) \right]. \] (3)

Here \( x_0^1 \) is the boundary of the matter distribution and \( T_0^0 \) are components of the energy–momentum tensor.

Let us also consider another RS for this same matter distribution, in which the metric has the form

\[ ds' = g_{00} (dx_0^0)^2 + g_{11} (dx_0^1)^2 + g_{22} [(dx_0^0)^2 + \sin^2 x^3 (dx_0^3)^2], \] (4)

where \( g_{00} = g_{00}(x^0, x^1) \), \( g_{11} = g_{11}(x^0, x^1) \), \( g_{22} = g_{22}(x^0, x^1) = -[x^1(x^0, x^1)]^2 \), and \( 0 < g_{00} < \infty, -\infty < g_{11} < 0, -\infty < g_{22} < 0 \), at least on the boundary of the \( R \) and \( T \) domains and in certain neighborhoods [i.e., the metric (4) has no singularities in the \( R-T \) boundary]. The coordinates introduced in such manner will be related to the previous coordinates by the relationships \( x^0 = x_0^0(x^0, x^1) \), \( x^1 = x_1^1(x^0, x^1) \), \( x^2 = x^2 \), \( x^3 = x^3 \), or in differential form:

\[ dx^0 = a^0_i dx_i, \quad a^0_i = \frac{\partial x^0}{\partial x_i}, \quad dx_0^1 = b^1_j dx_j, \quad i, j = 0, 1, b^1_j = \frac{\partial x_0^1}{\partial x_j}; \]

\[ dx^2 = dx^2, \quad dx^3 = dx^3. \] (5)

We shall assume \( x^2 = \text{const}, x^3 = \text{const} \).

2. Now, in the usual manner, let us introduce the definition of the magnitude of the velocity in the fixed and nonsingular RS:

\[ \bar{v}^2 = - \left( \mathfrak{g}_{11}/g_{00} \right) (dx^1/dx^0)^2, \quad \tilde{v}^2 = - \left( \mathfrak{g}_{11}/g_{00} \right) (dx^1/dx^0)^2, \] (6)

and also the relative velocities of the RS under consideration

\[ V^2 = v^2 \mid_{x^1=\text{const}}, \quad \tilde{V}^2 = v^2 \mid_{x^1=\text{const}} \] (7)

(the reduced velocities are chronometrically invariant [5, 6] and dimensionless; in order to go over to the "usual" velocities, the former must be multiplied by the speed of light). We ascribe the same sign as for the corresponding relation \( dx^1/dx^0 \) to all the velocities.

The definitions (7) and (6) permit obtaining

\[ V^2 = b^1_i b^1_j \tilde{v}^2, \quad \tilde{v}^2 = a^0_i a^0_j \bar{v}^2, \quad V^2 = \tilde{V}^2, \] (8)

for \( V^2 \) and \( \tilde{V}^2 \) when the transformations (5) and the orthogonality conditions of the coordinate lines are taken into account, where the velocities \( v, \bar{v}, V, \tilde{V} \) can be interrelated by means of

\[ \tilde{V}^2 (1 - V^2)(1 - \bar{v}^2) = (1 - \tilde{v}^2) \left[ \pm \tilde{v} \sqrt{(1 - \tilde{V}^2)(1 - \bar{V}^2) + \tilde{V} - \bar{v}} \right]. \] (9)