GEOMETRICAL CONNECTION BETWEEN THE EQUATIONS OF THE SPECIAL QUASICLASSICAL MODEL OF ATOMIC SYSTEMS

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We study the geometrical connection between the fundamental equations of the quasiclassical model proposed by Rodimov to explain the behavior of microparticles in atomic systems in his monograph "Self-Oscillating Quantum Mechanics." The v and u equations. It is shown that the geometry of the spaces for the v and u equations is different, but the metrical coefficients are related by simple algebraic relations.

In [1], starting from the assumption that each particle can be associated with a certain nonelectromagnetic field (the so-called K field) equations for the potential \(\kappa_u = (\kappa_0, \kappa_1)\) of this field were obtained in a special configuration space \(V_4\) (this space was introduced in [2]). It was also shown in [1] that in the nonrelativistic approximation the equation for the component \(\kappa_0\) of the K-field was the same as the equation introduced by Rodimov [3] to describe a special quantum field (this term taken from [3]) introduced to explain the behavior of microparticles in atomic systems.

The equation describing the quantum field was called the v-equation in the monograph [3]. According to Rodimov, the solution of the v-equation (subject to certain auxiliary conditions) yields energy levels for atomic systems which correspond to experiment.

Besides the v-equation, another equation (the u-equation) was introduced in [3]. This equation is interesting since the form of its spatial part is the same as the stationary Schrödinger equation and thereby implies a different interpretation of the physical meaning of the solutions (for the same boundary conditions) from the conventional probabilistic interpretation. Further, in the monograph [3] several qualitative arguments were advocated suggesting that there exists a geometrical connection between the v and u equations. However, in our opinion, this question, as treated in [3], is far from being completely resolved.

Therefore a more rigorous and complete study of the question of a geometrical connection between the v and u equations is of interest. The present paper is devoted to the solution of this problem.

According to [2], at each point \(p \in V_4\) (Minkowski space) one can set up an inertial frame with a velocity \(v_p = c \sqrt{\frac{\kappa_0}{g_{00}(\kappa_0)}}\), where \(g_{00}\) is the metrical coefficient of the special configuration space \(V_4\). The velocity \(v\) (the velocity of the inertial frame) must be measured with respect to a specified frame (we call it \(K_0\)) for which \(V_4\) is constructed. But in [4] we showed...
that if one starts from the postulate that the velocity of light is constant in all reference frames (we do not postulate in advance that the velocities of all reference frames are confined to the segment \([0, c]\)), then each point \(p \in V_4\) must correspond to not one, but two reference frames: one reference frame has a velocity \(v_p = c\sqrt{1 - \frac{g_{00}(x^0)}{c^2}}\) with respect to the system \(K_0\), and the other has the velocity

\[
u_p = \frac{c^2}{v_p}
\]

with respect to \(K_0\). Following [4], we will call this the conjugate frame (conjugate to the frame with velocity \(v_p\)).

Therefore, along with the space \(\tilde{V}_4\) assumed in [2], we can construct a four-dimensional space \(\tilde{V}_4\) with the metric

\[
d\tilde{S}^2 = \tilde{g}_{00}(x^i, t) c^2 dt^2 + \tilde{g}_{16} dx^i dx^6 = c^2 dt^2 \tilde{g}_{00}(x^i, t) [1 - \tilde{g}_{00}].
\]

where \(\tilde{g}_{ik}\) is the metric tensor of the space \(V_3\) (the spatial part of the Minkowski four-space \(V_4\)).

2. In [2] we constructed \(\tilde{V}_4\) as a fibered space represented as a set of isotropic surfaces \(\tilde{G}_{03}\) embedded in \(\tilde{V}_4\). In view of (1), each isotropic surface \(\tilde{G}_{03} \subset \tilde{V}_4\) corresponds to a surface \(\tilde{G}_3 \subset \tilde{V}_4\), given by the equation

\[
\tilde{g}_{00}(x^i, t) c^2 dt^2 + \tilde{g}_{16} dx^i dx^6 = c^2 dt^2 \tilde{g}_{00}(x^i, t) [1 - \tilde{g}_{00}].
\]

It is easily seen that along any line \(x^i = x^i(t)\) lying on the surface \(\tilde{G}_3 \subset \tilde{V}_4\),

\[
\tilde{g}_{00}(x^i(t), t) = \frac{u_1 u_1}{c^2}.
\]

We note that in this case the line \(x^i = x^i(t)\) is the classical trajectory of a particle [2] and therefore belongs to the isotropic surface \(\tilde{G}_{03} \subset \tilde{V}_4\). The surface \(\tilde{G}_3 \subset \tilde{V}_4\), satisfying the conditions (3) and (4), will be called conjugate to the surface \(\tilde{G}_{03} \subset \tilde{V}_4\).

3. The space \(\tilde{V}_4\) is also constructed as a fibered space, represented by a set of surfaces \(\tilde{G}_3\) (conjugate to the surfaces \(\tilde{G}_{03} \subset V_4\)) embedded in \(\tilde{V}_4\).

Therefore the absolute differential \(\tilde{V}(\cdot)\) in \(\tilde{V}_4\) is defined by relations formally identical to those for the absolute differential \(V(\cdot)\) in \(V_4\) [2], and differing only by the fact that the quantities \(\tilde{G}_{03}\) in \(\tilde{V}(\cdot)\) are changed into the quantities \(\tilde{G}_3\). Obviously this definition of the absolute differential ensures parallel transfer along any path on \(\tilde{G}_3 \subset \tilde{V}_4\). The space \(\tilde{V}_4\) constructed in this way will be referred to as conjugate to the space \(\tilde{V}_4\).

4. Using the results of Sections 2 and 3, it is not difficult to show that the classical trajectories of particles in a potential field with respect to a given inertial frame can be represented as geodesics lying on the surface \(\tilde{G}_3 \subset \tilde{V}_4\). In order for this to be true, \(\tilde{S}_{00}\) (see [2] ) must be chosen in the form

\[
\tilde{S}_{00} = |\partial, \ln ((1 - \tilde{g}_{00}) \tilde{g}_{00})|/2.
\]

Then the equation of motion of a test particle takes the form

\[
Dp^a = - \tilde{g}_{00} p^a dx^0 (p^m = m dx^m/d\tau),
\]

where \(d\tau = \sqrt{1 - \tilde{g}_{00}(x^i(t), t) dt}\).

5. Now it is clear that in order to establish a geometrical connection between \(V\) and \(u\) equations, as formulated in the Introduction, we must assume that any particle corresponds to not only the \(K\) field, but also to another nonelectromagnetic field (we call it the \(S\) field) whose description in the space \(\tilde{V}_4\) is formally equivalent to the description of the \(K\) field in the space \(\tilde{V}_4\) (conjugate to \(\tilde{V}_4\)).