GENERALIZED CAUSALITY CONDITION IN QUANTUM FIELD THEORY WITH TORSION

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The Schwinger–De Witt method of local expansion of the effective action is applied to a model vector field theory which satisfies a generalized causality condition. One-loop divergences are computed. It is shown that the theory is non-renormalizable. The effective potential of the theory has a local minimum with a nonzero average value of the vector field, stable with respect to small slowly varying perturbations.

When one quantizes fields in the Einstein–Cartan theory or in a theory with propagating torsion, one encounters Lagrangians in which the quadratic part, unlike the self-interaction, is invariant under local gauge transformations. The specifics of the problem are that in such theories one cannot use the perturbation theory around empty space, while physical perturbations propagate along several "light" cones. These cones are defined with respect to a set of reversible matrices playing the role of several different space–time metrics. It was suggested in [1] that the Schwinger–De Witt techniques [2, 3] of local expansion of the quantum effective action can be generalized to such theories, which would satisfy a generalized causality condition. Using such techniques becomes unavoidable due to the inapplicability of the perturbation theory on the background of flat and empty space.

Here we shall consider the example of the theory of a vector field $A_{\mu}$ whose action*

$$S = \int d^4x g_{\mu\nu} \left[ -\frac{1}{4} (\nabla_{\mu} A_{\nu} - \nabla_{\nu} A_{\mu})^2 + \frac{3}{4} (g^{\mu\nu} A_{\mu} A_{\nu})^2 \right]$$

models the above properties, characteristic for the vector field in the Einstein–Cartan theory [4, 5] or for conformally invariant theory with propagating torsion induced by radiative corrections [6, 7].

Due to the invariance of the quadratic part of Eq. (1) under the transformations $A_{\alpha} \rightarrow A_{\alpha} + \alpha f$, the inverse propagator of the theory

$$D^{a\beta}(V) = (g^{a\beta} V_{a} V_{\beta})^{-1} = V_{a} V_{\alpha} - 2 (A^{a})^2 G_{\alpha\beta},$$

$$A^{a} = g^{a\mu} A_{\mu}, \quad G^{a\beta} = A^{-2} (g^{a\beta} + 2 A^{-2} A^{a} A^{\beta})$$

is nondegenerate only for nonzero $A_{\alpha}$. In this case, the generalized causality condition [1]

$$\det D^{a\beta}(i\omega) = g^{-1/2} (a A^{2}) (g_{\mu\nu} n_{\mu} a_{\nu} + a A^{2}) (A^{a} G^{a\beta} n_{\mu} a_{\nu} + 3 a A^{2})$$

*We denote $R = g^{\mu\nu} R_{\mu\nu} = g^{\mu\nu} (3 \alpha_{a}^{\alpha_{a}} \gamma_{\alpha_{a}}^{\alpha_{a}} - \ldots)$, sign $g_{\mu\nu} = +2$, $g = |\det g_{\mu\nu}|$, $\nabla_{\mu}$ is the covariant derivative with respect to $g_{\mu\nu}$, $\alpha$ is the constant parameter of the theory.


(D^{AB}(\nu)) is a finite-dimensional matrix obtained from D^{A}(\nu) by the substitution of \nu by a numerical vector in_\mu) defines two particles of mass m propagating in the background metric g^{\nu}(1) = g^{\nu\mu} with momentum n_\mu, satisfying equation\(g^{\nu}(1) n_\mu n_\mu + m^2 = 0\) (\(m^2 = \lambda A^2\)). and a particle of mass \(m/3\), propagating in the metric g^{\nu}(2) = A^2 g^{\nu\mu}, g^{\nu}(2) n_\mu n_\mu + 3m^2 = 0. Therefore, like the linear massive vector field [3], the theory (1) has only three dynamical modes out of four components of \(A_\mu\).

The theory (1) is conformally invariant in 4-dimensional space. The part of the action (1) quadratic in field perturbations \(h_\alpha\) can be rewritten in the explicitly conformally invariant form:

\[
S_2 = \frac{1}{2} \int d^4x (\tilde{g})^{1/2} \tilde{h}_\alpha \left[ S^{(\alpha)}(\overline{\nu}) - \overline{\nu}^{\alpha} \overline{\nu} \right] h_\alpha,
\]

where the two metrics \(\tilde{g}_{\mu\nu}\) and \(G_{\mu\nu}\) are invariant under conformal transformations of \(g_{\mu\nu}\) and \(A_\mu\) while all quantities, operators, and covariant derivatives \(\overline{\nu}_\alpha\) and \(\nu_\alpha = \tilde{g}^{\alpha\nu} \tilde{g}_{\nu\beta}\) in Eqs. (5)-(7) are defined with respect to the metric \(\tilde{g}_{\mu\nu}\). To make the computations simpler, it is convenient to convert the theory (5) into a gauge theory in which all field variables are dynamical. Parametrizing the field \(h_\alpha\) in terms of the new fields \(\eta = (f, \eta_\mu)\), \(h_\alpha = a_\alpha + \partial_\alpha f\), the action (5) becomes invariant under the gauge transformations \(\eta \rightarrow \eta + \Delta \eta: \Delta a_\alpha = \partial_\alpha \epsilon, \Delta f = - \epsilon\). Choosing the gauge-fixing term to be explicitly conformally invariant:

\[
S_{GB} = \frac{1}{2} \int d^4x (\tilde{g})^{1/2} (\tilde{\nu}^2 a_\alpha)^2,
\]

one can reduce the general quadratic action to the form

\[
S_2 + S_{GB} = \int d^4x (\tilde{g})^{1/2} F_\alpha F_\alpha,
\]

\[
F = F_\phi + V,
\]

\[
F_\phi = \text{diag} (\lambda G^\nu D_\mu D_\nu, S^{(\nu)}(\overline{\nu})),
\]

\[
V = \begin{bmatrix}
0 & \lambda G^\nu \overline{\nu}_\nu \\
-\lambda G^\nu \overline{\nu}_\nu & 0
\end{bmatrix}
\]

where the matrix operator \(F\) acts on the column consisting of the scalar field \(f\) and vector field \(a_\mu\), and \(D_\mu\) is the covariant derivative with respect to the metric \(G_{\mu\nu}\). The local expansion [1-3] of the one-loop effective action of the gauge theory (8), (9)

\[
i\mathcal{W} = -\frac{1}{2} \text{Tr} \ln F + \text{Tr} \ln [\tilde{g}^{\alpha\nu} \tilde{\nu}_\alpha \tilde{\nu}_\nu]
\]

(\(\text{Tr} \ln [\tilde{g}^{\mu\nu} \tilde{\nu}_\mu \tilde{\nu}_\nu]\) is the ghost contribution) can be achieved by expanding \(\text{Tr} \ln F\) in powers of the perturbation \(V\). The subsequent terms of this expansion contain one-loop divergences

\[
i\mathcal{W}^{\text{div}} = \left\{ -\frac{1}{2} \text{Tr} \ln [G^{\alpha\nu} D_\mu D_\nu] - \frac{1}{2} \text{Tr} \ln S^{(\nu)}(\overline{\nu}) + \right.

\left. + \text{Tr} \ln [\tilde{g}^{\alpha\nu} \tilde{\nu}_\alpha \tilde{\nu}_\nu] + \frac{1}{4} \text{Tr} [(VF_\phi^{-1})^4] + \frac{1}{8} \text{Tr} [(VF_\phi^{-1})^6] \right\}
\]

The first three terms in Eq. (14) can be computed by the direct application of the Schwinger–De Witt algorithm [2] to the minimal vector operator (6) with the space–time metric \(g_{\mu\nu}\) and to the minimal scalar operators \(G^{\mu\nu} D_\mu D_\nu\) and \(\tilde{g}^{\mu\nu} \tilde{\nu}_\mu \tilde{\nu}_\nu\) with the metrics \(G_{\mu\nu}\) and \(\tilde{g}_{\mu\nu}\) accordingly.

The peculiarity of the computation of the 4th and 5th terms is that the Green function of the operator (11)

\[
F_\phi^{-1} = \text{diag} \left( \frac{1}{\lambda} H(D), H^{(\overline{\nu})} \right)
\]