MORE ON THE \( \tau\mu e \) PROBLEM: THE PROCESS \( e^+e^- \rightarrow \ell^+\ell^- \) IN THE REGION OF THE Z-RESONANCE

T. M. Aliev, S. F. Sultanov, and R. Sh. Yakh'yaev

Theoretical and experimental aspects of the study of the \( e\mu\tau \) problem in effects of neutral weak currents in colliding \( e^+e^- \) beams in the region of the Z resonance are discussed. Quantities that describe the 'degree' of violation of \( e\mu\tau \) universality are analyzed. In particular, it is shown that by measuring the spin asymmetry for angles \( \theta \sim 160^\circ \) it is possible to determine the degree of violation of \( \ell\mu \) universality \( \Delta = g^\ell\mu_A - g^\ell\mu_V \) to an accuracy of up to \( \Delta \sim 0.01 \).

With the discovery of the \( \tau \) lepton, the \( \mu\tau \) problem [1] has become the \( \tau\mu \) problem or, if we assume quark–lepton symmetry, the problem of fermion 'families.' It is well known that \( \tau\mu \) universality of weak charged currents (WCCs) has been checked to high accuracy (although this does not preclude the possibility of small deviations from the \( \tau\mu \) universality of WCCs). As far as the neutral weak currents (NWCs) are concerned, the situation here is less definite. For example, the problem to what extent \( \tau\mu \) universality is realized indirectly in nature reduces to the accurate study of the structure of NWCs, i.e., to the exact determination of the vector and axial weak-interaction constants.

A unique opportunity for studying the structure of NWCs will appear in the region of the Z resonance, which will become accessible in the near future with the startup of the SLC accelerator (with \( \sqrt{s} \sim 100 \) GeV). In connection with this, research on the effects responsible for possible violations of \( \tau\mu \) universality in the region of the Z resonance is of particular interest. This paper is devoted to a discussion of this subject in the process \( e^+e^- \rightarrow \ell^+\ell^- \) (\( \ell = \mu, \tau \)).

The differential cross section of the process \( e^+e^- \rightarrow \ell^+\ell^- \) in the region of the Z resonance has the form

\[
\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{16m_e^2} |s_{2e}|^2 \left\{ \left[ \left( g_{\theta}^\ell \right)^2 + \left( g_{\alpha}^\ell \right)^2 \right] \left( \left( g_{\theta}^e \right)^2 + \left( g_{\alpha}^e \right)^2 \right) (1 + \cos^2\theta) + \right. \\
+ 8g_{\theta}^e g_{\alpha}^\ell g_{\alpha}^\ell g_{\theta}^\ell \cos \theta \right\} - \left[ \left( g_{\theta}^\ell \right)^2 - \left( g_{\alpha}^\ell \right)^2 \right] (g_{\theta}^e)^2 + (g_{\alpha}^e)^2 \right\} \lambda_1 \lambda_2 \sin^2\theta \cos^2\theta \}
\]

in the case of transversely polarized colliding beams;

\[
\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{16m_e^2} |s_{2e}|^2 \left\{ \left[ \left( g_{\theta}^\ell \right)^2 + \left( g_{\alpha}^\ell \right)^2 \right] \left( \left( g_{\theta}^e \right)^2 + \left( g_{\alpha}^e \right)^2 \right) (1 + \cos^2\theta) + \right. \\
+ 8g_{\theta}^e g_{\alpha}^\ell g_{\alpha}^\ell g_{\theta}^\ell \cos \theta \right\} (1 - \lambda_1 \lambda_2) - 2 \left[ \left( g_{\theta}^\ell \right)^2 + \left( g_{\alpha}^\ell \right)^2 \right] g_{\theta}^e g_{\alpha}^\ell \cos^2 \theta + g_{\theta}^e g_{\alpha}^\ell \left[ \left( g_{\theta}^e \right)^2 + \left( g_{\alpha}^e \right)^2 \right] (1 + \cos^2\theta) \right\} (l_1 - l_2)
\]

in the case of longitudinally polarized colliding beams; and

---

LITERATURE CITED

In expressions (1)-(3)

\[ |s_z^{(e)}|^2 = \frac{1}{4x_b(1-x_w)^2} \left( \frac{m_Z}{1-z} \right)^2 \]  

when \( p = (m_H/m_\omega \cos \theta)^2 = 1 \), and \( g_V^e, g_V^\tau, g_A^e \), and \( g_A^\tau \) are the vector and axial constants for the electron current and the \( Z(\ell = \mu, \tau) \) current. In the derivation of expressions (1)-(3), summation was performed over the polarizations of the final particles, and the assumption was made that \( \beta_\ell = 1 \).

Let us also present the expression for the total cross section of the process \( e^+ + e^- \rightarrow Z^+ + Z^- \) in the region of the \( Z \) resonance

\[ \sigma(t_1, t_2) = \frac{\pi x^2}{3 m_Z^2} |s_z^{(e)}|^2 \left[ (g_V^e)^2 + (g_A^e)^2 \right] \left[ (g_V^\tau)^2 + (g_A^\tau)^2 \right] \left( t_1 - t_2 \right) \]  

(5)

\[ \sigma(t_1, t_2) = \frac{\pi x^2}{3 m_Z^2} |s_z^{(e)}|^2 \left[ (g_V^e)^2 + (g_A^e)^2 \right] \left[ (g_V^\tau)^2 + (g_A^\tau)^2 \right] \left( t_1 - t_2 \right) \]  

(6)

\[ \sigma(0, 0) = \frac{\pi x^2}{3 m_Z^2} |s_z^{(e)}|^2 \left[ (g_V^e)^2 + (g_A^e)^2 \right] \left[ (g_V^\tau)^2 + (g_A^\tau)^2 \right] \]  

(7)

It should be noted that expressions (1)-(7) also are applicable to the process \( e^+e^- \rightarrow e^+e^- \), since the contribution of exchange diagrams is negligibly small in the region of the \( Z \) resonance (in this process \( g_V^{e, A} = g_V^{e, A} \)).

Let us turn to the investigation of the possibilities for studying the structure of NWCs in the region of the \( Z \) resonance. First let us note that the study of the partial decay widths of the \( Z \) boson via the channels \( Z \rightarrow \ell^+\ell^- \) (\( \ell = e, \mu, \tau \)) makes possible the determination of the combination \((g_V^e)^2 + (g_A^e)^2) \) [2]. For \( \ell = e \) and \( \mu \) this can be determined (experimentally) to quite high accuracy, whereas the situation is less auspicious for the \( \tau \) lepton because of the relatively low efficiency of detection of the \( \tau \) lepton from decay products. Let us note that careful examination of the radiation corrections for these processes is necessary. Hence, the measurement of \( Br(Z \rightarrow \ell^+\ell^-) \) (for more detail, see [3]) can serve as a good test for checking the \( \mu-e \) universality of the combination \((g_V^e)^2 + (g_A^e)^2) \). In this paper we investigate other possibilities for checking the \( \tau-\mu-e \) universality of the NWCs of charged leptons, which follow from an analysis of the differential cross sections (1)-(3) and total cross sections (5)-(7) of the processes \( e^+e^- \rightarrow Z^+Z^- \) (\( \ell = e, \mu, \tau \)) in the region of the \( Z \) resonance.

As we can see from expressions (1) and (3), in the case of transversely polarized and unpolarized initial beams, additional information on NWC structure can be obtained by studying the asymmetry in the angular distribution of the final lepton

\[ A = \frac{\sigma(\theta)/d\Omega - \sigma(\pi - \theta)/d\Omega}{\sigma(\theta)/d\Omega + \sigma(\pi - \theta)/d\Omega} \]  

(8)

which has the form (integration is performed over the azimuthal angle \( \varphi \))

\[ A = \frac{8g_V^e g_A^e \theta}{((g_V^e)^2 + (g_A^e)^2)(1 + \cos^2 \varphi)} \]  

(9)

In the case of longitudinal polarization of the initial beams, the NWC structure can be studied by investigating:

a) the asymmetry in the angular distribution of the final lepton

\[ A = \frac{g_V^\ell g_A^\ell}{(g_V^\ell)^2 + (g_A^\ell)^2} \cdot \frac{2g_V^\ell g_A^\ell(1 - t_1 t_2) - ((g_V^\ell)^2 + (g_A^\ell)^2)(t_1 - t_2)}{((g_V^\ell)^2 + (g_A^\ell)^2)(1 - t_1 t_2) - 2g_V^\ell g_A^\ell(t_1 - t_2)} \cdot \cos \theta \]  

(10)

b) the beam-polarization effect, e.g., when \( t_2 = 0 \)

\[ N = \frac{\sigma(0, 0)/d\Omega - \sigma(t_1, 0)/d\Omega}{\sigma(0, 0)/d\Omega + \sigma(t_1, 0)/d\Omega} = \frac{c_1 t_1}{c_1 - c_1 t_1} \]  

(11)