APPLICATION OF RELATIVISTIC COULOMB GREEN'S FUNCTION TO THE CALCULATION OF TRANSITION PROBABILITIES IN AN ELECTRIC FIELD

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The probability of transitions between the levels of multiply charged ions in a strong electric field are expressed in terms of the matrix elements of the relativistic Coulomb Green's function. The probability of magnetic dipole decay of the 2s\(^{1/2}\) level of a hydrogenlike ion in a field \(F = 10^7\) V/cm is calculated.

In recent years, much attention has been devoted to experimental and theoretical investigations into the spectra of multiply charged ions. Such spectra are observed in the solar corona as well as under special laboratory conditions. The multiply charged ions can then be in a strong electric field, whose influence on the energy levels was considered in [1]. The crossing in an external field of levels with different parity can be used to study parity violation effects in atomic systems [2, 3].

It is of considerable interest to take into account the influence of an external field on the probabilities of transitions between the levels of multiply charged ions. For example, the radiative decay of 2s\(^{1/2}\) levels of hydrogenlike ions in an electric field has been studied in connection with measurement of the Lamb shift [4]. In ions with high multiplicity of the ionization (\(Z - N \gg 1\), where \(Z\) is the charge of the nucleus and \(N\) is the number of electrons) the relativistic effects become very appreciable [5], so that in the calculation of the energy levels and the transition probabilities it is necessary to take into account fully relativistic theory without an expansion in the parameter \(\alpha Z\) (\(\alpha\) is the fine structure constant).

In the present paper, we consider the probabilities of transitions between the levels of multiply charged single-electron ions in the first order of perturbation theory in the interaction with a homogeneous electric field of intensity \(F\) (which may reach \(10^9\) V/cm). We calculate the probability of magnetic dipole decay of the 2s\(^{1/2}\) level, which is forbidden in the nonrelativistic limit.

For the probability \(W_{AB}\) of the transition between the states \(A\) and \(B\), we use its expression in terms of the S-matrix element, as in [6]:

\[
W_{AB} = 2\pi \left| \langle B | M | A \rangle \right|^2, \quad \langle B | S | A \rangle = -2\pi i \langle B | M | A \rangle \delta (E_A^0 - E_B^0),
\]

where \(E_{A,B}^0\) are the unperturbed energies of the states \(A\) and \(B\). Here and in what follows, we use a system of units in which \(\hbar = c = m = 1\) (\(m\) is the electron mass).

In calculating the transition probability in the first order of perturbation theory in the interaction with the external field, we must take into account the contribution from the diagrams in Fig. 1. Here, the wavy line denotes the emitted photon, the broken line the photon absorbed from the external field, while the double line represents the electron propagator. In a completely relativistic calculation, one must from the very beginning use the Dirac wave functions:

\[
h (r) \psi_{njl} = E_n \psi_{njl}.
\]

Here, \(h(r)\) is the relativistic single-electron Dirac Hamiltonian for the electron in the field of the nucleus, \(n\) is the principal quantum number, \(j\) and \(m\) are the total angular momentum of the electron and its projection, and \(l\) is the orbital quantum number.

The interaction with the homogeneous external electric field mixes states with different values of the orbital quantum number, so that as the wave functions of the
initial, A, and final, B, states we must use functions of the form
\[ \psi_{nlm} = \sum_i a_i(n^i) \phi_{nlm}. \]  
(3)

The mixing coefficients \(a_i(n^i)\) are obtained by diagonalizing the Hamiltonian which takes into account the interaction with the external field:
\[ H(r) = h(r) + a_{1/2} F_z, \]
(4)
where \(z\) is the Cartesian coordinate of the electron in the direction of the field.

In the calculation of the S-matrix element (1) we use the exact relativistic expression for the operator of the interaction with the electromagnetic field [7]. The summation over the virtual states in the matrix elements corresponding to the diagrams in Figs. 1b and 1c is done by means of the relativistic Coulomb Green's function \(G_E(r_1, r_2)\) of the Dirac equation [8]. It is here necessary to eliminate intermediate states identical to the initial state, i.e., to use the reduced Green's function
\[ G_{E_{nj}}(r_1, r_2) = \lim_{E \to E_{nj}} \left[ G_E(r_1, r_2) - \sum_m \frac{|\psi_{nlm} > < \psi_{mlm}|}{E_{nj} - E} \right]. \]
(5)

The probabilities of radiative transitions in single-electron ions in the absence of an external field (the matrix element corresponding to the diagram in Fig. 1a) are calculated in [9].

The S-matrix element corresponding to the diagrams in Figs. 1b and 1c is
\[ V_{\lambda\nu}(A, B) = i2\pi \left[ < \psi_B(r_2)|z_2G_{EB}(r_2, r_1)(\gamma A_{1\nu}(r_1))|\psi_A(r_1)> + < \psi_B(r_2)|\gamma A_{1\nu}(r_2)G_{EB}(r_2, r_1)\gamma z_1|\psi_A(r_1)> \right]. \]
(6)

Here, \(\lambda\) is the multipolarity of the transition, \(A_{1\nu}(r)\) is the corresponding vector potential of the electromagnetic field [7], and \(\gamma_\lambda\) are the Dirac matrices.

Using the expression (5) for the reduced Green's function and going over to the quadratic relativistic Coulomb Green's function \(G_E(r_1, r_2)\) [8], we can write the expression for \(V_{\lambda\nu}(A, B)\) in the form
\[ V_{\lambda\nu}(A, B) = i2\pi \lim_{E_B \to -E_A} \left[ \frac{1}{2} < \psi_B(r_2)|(\gamma_\lambda z_\nu + z_\lambda \gamma_\nu) \left( \frac{2az}{r_2} + E_i + E_B \right) |\psi_A(r_1)> \right]. \]
(7)

Using the expression (3) for the wave functions of the states A and B, we can write
\[ V_{\lambda\nu}(A, B) = \beta 2\pi \sum_{l^2m^2} a_{\lambda l m}^* (n_1 l_1 m_1) a_{\nu l m} (n_2 l_2 m_2) V_{1\nu}(n_1 l_1 m_1, n_2 l_2 m_2). \]
(8)

The states A and B are determined by the sets of quantum numbers \(n_0 m_0 n_2 l_2 m_2\) and \(n_1 j_1 m_1\), respectively. Using the partial-wave expansion of the Green's function [8] and the following abbreviated form of expression for the potential of the electromagnetic field [7]:

**TABLE 1**

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Fig. 1