Using 2-dimensional quantum chromodynamics as an example, it is shown that the
gauge-invariant fields formalism allows one to avoid the difficulties related to
the choice of an infrared regularization, and leads to confinement of both single
quarks and quark-antiquark pairs.

The color confinement problem in quantum chromodynamics is intimately related to studying
the behavior of total Green functions in the infrared area. One of the possible approaches to
this problem is using the $1/N$-expansion (see, e.g., the review [1]). One finds that the ob-
tained results strongly depend on the choice of the method of infrared regularization [2, 3].
Such a situation has been previously known to exist in 2-dimensional quantum chromodynamics
(QCD$_2$). In the pioneer paper [4] the role of an infrared regularizer was played by the
function $\Theta(k_\perp - \lambda)$, which cuts off the momentum integration at $k_\perp < \lambda$. Then the infrared regu-
larizer $\lambda$ was pushed to zero, which ensured quark confinement in terms of the spinor Green
function.

A different result was obtained by the authors of [5], whose proposal consisted in under-
standing the infrared singularity in the sense of principal value. The quark propagator found
in [5] possesses a singularity at the finite point $k^2 = m^2 - g^2 N/\pi$.

Then in [6] it was shown that an infrared regularization using a $\theta$-function, contradicts
the requirement of gauge invariance and violates Ward identities. Difficulties related to
violation of gauge invariance were also mentioned in [7]. The regularization proposed in [6],
leads to the quark propagator having a cut in the complex $p^2$ plane, which does not have a
clear physical interpretation.

In the present paper we show that the difficulties related to the choice of infrared regu-
larization can be overcome by using a gauge-invariant approach developed in the series of
papers [8]. It appears that the gauge-invariant approach leads to a uniquely defined gluon
propagator which does not require an additional infrared regularization. From the conceptual
point of view, this paper continues the study of QCD$_2$ started in [11].

Let us introduce gauge-invariant field variables. Let $A_\mu(x)$ be the initial gluon field
whose field strength is

$$G_{\mu\nu}(x) = \partial_\mu A_\nu(x) - \partial_\nu A_\mu(x) + ig[A_\mu(x), A_\nu(x)].$$  \hspace{1cm} (1)

Under gauge transformations $U(x)$, the gluon field and the field strength tensor (1) transform
as follows:

$$A_\mu(x) \rightarrow U(x) A_\mu(x) U^{-1}(x) - (i/g) U(x) [\partial_\mu U^{-1}(x)];$$  \hspace{1cm} (2)

$$G_{\mu\nu}(x) \rightarrow U(x) G_{\mu\nu}(x) U^{-1}(x).$$  \hspace{1cm} (3)

Let us go over to new fields in the QCD$_2$ Lagrangian with the aid of the gauge transfor-
mation

$$V(\xi; x) = P \exp\left[i \int \frac{d\eta}{\xi} A_\mu(x)\right].$$  \hspace{1cm} (4)

where $\xi$ is some fixed space point, and $P$ denotes ordering of operators along the integration

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path connecting the points $\xi$ and $x$. The resulting Lagrangian will be written in terms of the new vector fields $B_\mu(x|\xi)$, which under gauge transformations of the initial field $A_\mu$ is transformed by the laws of [8]:

$$B_\mu(x|\xi) \rightarrow U(\xi)B_\mu(x|\xi)U^{-1}(\xi).$$  (5)

A similar global transformation law holds for the field strength tensor $G_{\mu\nu}(x|\xi)$ built with the aid of the new fields $B_\mu(x|\xi)$:

$$G_{\mu\nu}(x|\xi) = V(\xi; x)G_{\mu\nu}(x) V^{-1}(\xi; x) \rightarrow U(\xi)G_{\mu\nu}(x|\xi)U^{-1}(\xi).$$  (6)

Note that when $\xi \rightarrow \infty$, and if the condition $\lim_{\xi \rightarrow \infty} U(\xi) = 1$ holds, in accordance with Eqs. (5) and (6) we obtain a formulation of the theory in terms of gauge-invariant field variables. Besides, the limit transition $\xi \rightarrow \infty$ restores the translational invariance of the Green function violated at the fixed finite value of $\xi$. Thus a natural limit procedure emerges in our approach, namely

$$\xi_\mu = a_{\eta_\mu}, \quad A \rightarrow \infty,$$  (7)

after which the theory can be formulated in terms of gauge-invariant field variables, and the translational invariance of the Green function gets restored.

In the case of a rectilinear contour in Eq. (4), the fields $B_\mu(x|\xi)$ satisfy the Fock gauge condition

$$(x-\xi)^a B_a(x|\xi) = 0$$  (8)

and can be expressed via the field strength tensor with the aid of the inversion formula

$$B_\mu(x|\xi) = \int d\xi(x-\xi)^a G_{\mu a}(\xi + z(x-\xi)|\xi).$$  (9)

The existence of linear inversion formulas, where vector fields are expressed via the field strength tensor, is a characteristic feature of the approach under consideration (see also [8-12]).

In the framework of $1/N$-expansion, one uses the free gluon propagator as the kernel in the Dyson–Schwinger equation for the quark propagator and in the Bethe–Salpeter equation for the vertex function. To compute the free gluon propagator, let us use the fact that in the free case, the vacuum average of a chronological product of field strength tensor in two dimensions can be easily shown to have the following form:

$$\langle 0| T G_{\mu\nu}(x) G_{\rho\sigma}(y) | 0 \rangle = \epsilon_{\mu\nu\rho\sigma}(x-y).$$  (10)

where $\epsilon_{\mu\nu}$ are the components of the antisymmetric tensor ($\epsilon_{00} = \epsilon_{11} = 0; \epsilon_{01} = - \epsilon_{10} = 1$).

Using the expression (10) and the inversion formulas (9), we can find the propagator of the vector field $B_\mu(x|\xi)$:

$$D_\mu(x, y|\xi) = \int d\xi(x-\xi)^a G_{\mu a}(\xi + z(x-\xi)|\xi).$$  (11)

where for every two-dimensional vector $z_\mu$ we introduce the notation

$$\tilde{z}_\mu = \epsilon_{\mu\nu}z^\nu.$$  (12)

It is easy to see that the propagator (11) has the property*

$$D_\mu(x, y|\xi) = D_\mu(x-\tilde{z}, y-\tilde{z}|0).$$  (13)

The expression (11) is an explicitly covariant form of the propagator previously obtained in [11].

In the leading order of $1/N$ expansion, the equation for the quark propagator in the coordinate representation has the form

$$\hat{M}(x, y|\xi) = -i g^2 D_\mu(x, y|\xi) [\gamma^\mu \hat{G}(x, y|\xi) \gamma^\nu],$$  (14)

*The properties of vector fields in the Fock gauge under translations were studied in [9].