DYNAMIC TREATMENT OF GRAVITATION. THE INTERACTION EQUATION AND EQUATIONS OF THE GRAVITATIONAL FIELD

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Commencing from a traditional dynamic interpretation of gravitation based on the work of Gupta and Thirring, the problem of constructing a contradiction-free theory of the gravitational field is solved without recourse to a curved space. Self-action is introduced into the Lagrangian without disrupting the law of conservation of energy.

INTRODUCTION

Over the entire course of development of Einstein's general theory of relativity alternative theories of gravitation of various types have been proposed, among which we may distinguish those using a planar space-time, based on field representations (Gupta [1], Thirring [2], Feynman [3], Kibble [4], and Tonnelat [5]). Such theories are based on the concepts, still valid in the general theory of relativity, that the source of the gravitational field is the energy-momentum density tensor $T_{uv}$, while the gravitational field potential is given by a symmetric second rank tensor $h_{uv}$ (where the Greek letter subscripts take on the values 0, 1, 2, 3). In the 1960's development of alternative gravitational theories in a planar space produced no solution capable of competing with the general theory, as a result of which the opinion developed that "both approaches to gravitation - the methods of curved and planar space - lead to a single physical theory" [4] (see also [6]).

Logunov, Loskutov, and Mestvirishvili [7] recently subjected the general theory of relativity to a new critique and demonstrated a new approach to introduction of planar space. The basis of their "relativistic theory of gravitation" is a "principle of geometrization, according to which the equation of motion of matter under the influence of a gravitational field $h_{uv}$ in a Minkowsky space with metric tensor $\gamma_{uv}$ can be identically represented as equations of motion of matter in a Riemann space-time with metric tensor $\delta_{uv}$ dependent on the gravitational field $h_{uv}$ and the metric tensor $\gamma_{uv}$.

The present study will maintain the viewpoint that a gravitational theory should be constructed on the basis of field concepts, although in contrast to the relativistic theory of gravitation, the salient point will be determining the essence of gravitational interaction, which will then permit a systematic tracing of the dynamics of that interaction [8].

In contrast to geometrical treatments, a dynamic treatment of the gravitational field does not, at least not initially, relate the gravitational field potential $h_{uv}$ to the metric tensor $\gamma_{uv}$. The Lagrangian formalism will be developed against the background of a Minkowsky space with arbitrary coordinates

$$ds^2 = \gamma_{uv}dx^udx^v.$$

The basic problem of a dynamic gravitational theory is to consider the specific self-action of the gravitational field. The physics behind this self-action involves the fact that being possessed of energy, the gravitational field generates itself. Traditional attempts to solve this problem [1, 2, 4] met with difficulties, since introduction of self-action disrupts the law of conservation of energy. It has been possible to overcome these difficulties and construct a theory which simultaneously considers field self-action fully and maintains a conservative energy tensor, by using a method in which the Lagrangian is a solution of some interaction equation and does not have a prespecified fixed structure expressed in terms of the potential and its derivatives.


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INTERACTION EQUATIONS

We will assume that the interaction of the gravitational field with other fields and itself can be described by an interaction Lagrangian of the form

\[ L_{\text{int}} = \kappa h_{\mu\nu} T^\mu_\nu \]  

(1)

and that the complete Lagrangian is the sum of the interaction Lagrangian and a seed Lagrangian \( L_0 \), which decomposes into two parts, the gravitational field Lagrangian \( L_0^{\text{grav}} \) and the nongravitating material Lagrangian \( L_0^{\text{mat}} \):

\[ L = \kappa h_{\mu\nu} T^\mu_\nu + L_0^{\text{grav}}(h_{\phi;\mu}) + L_0^{\text{mat}}(\Phi^A, \Phi^A;\phi), \]  

(2)

where \( \Phi^A \) is the set of field potentials, \( \kappa \) is the interaction constant, \( T^\mu_\nu \) is the energy tensor of all fields and their interaction energies, the comma denotes a covariant derivative in a Minkowsky space with arbitrary coordinates; \( L_0^{\text{mat}} \) are standard field Lagrangians constructed without consideration of gravitation, \( L_0^{\text{grav}} \) is the Lagrangian of the primary gravitational field, which is assumed fundamentally linear.

The source of the interaction is the energy tensor, which in the present treatment plays the role of source flux density. The unique feature of gravitation is that the gravitational field is its own source, which in the final reckoning leads to nonlinearity of the gravitational field equations. The universality of gravitation rests upon the universality of interaction Lagrangian (1), which is identical for all fields, including the gravitational.

Since the energy tensor \( T^\mu_\nu \) itself is defined in terms of Lagrangian (2), the expression for Lagrangian (2) can be transformed into an equation for the Lagrangian, which we will term the interaction equation. Taking the canonical energy tensor for a base,* we obtain the explicit form of the interaction equation:

\[ \kappa h_{\mu\nu} \Phi^A = \frac{\partial L}{\partial \Phi^A} + \kappa h_{\mu\nu} \frac{\partial L}{\partial h_{\phi;\mu}^A} - (1 + \kappa h_{\mu}^A) L = -L_0^{\text{mat}} - L_0^{\text{grav}}, \]  

(3)

The interaction equation is the basic core of the dynamic treatment of gravitation - its solution will be the complete Lagrangian of the field system, in which correct description of field interaction is insured, and on the basis of which an energy tensor will be constructed which fully considers the energy of the fields and their interaction while simultaneously satisfying the conservation law. This solution is nontraditional in that the complete Lagrangian cannot be written in the form of a polynomial in \( h_{\mu\nu} \), but is written in tensor form.

The solution of Eq. (3) consists of two terms: \( L = L^{\text{mat}} + L^{\text{grav}} \); \( L^{\text{mat}} \) is the Lagrangian of the fields \( \Phi^A \), considering their gravitational interaction, defined from the equation

\[ \kappa h_{\mu\nu} \Phi^A = \frac{\partial L^{\text{mat}}}{\partial \Phi^A} - (1 + \kappa h_{\mu}^A) L^{\text{mat}} = -L_0^{\text{mat}}; \]  

(4)

\( L^{\text{grav}} \) is the Lagrangian of the gravitational field, which considers the self-action of the field, in contrast to the seed Lagrangian \( L_0^{\text{grav}} \). That portion of interaction equation (3) which defines \( L^{\text{grav}} \):

\[ \kappa h_{\mu\nu;\rho} \frac{\partial L^{\text{grav}}}{\partial h_{\mu\nu;\rho}} - (1 + \kappa h_{\mu}^A) L^{\text{grav}} = -L_0^{\text{grav}}, \]  

(5)

will be termed the equation of gravitational field self-action.

GRAVITATIONAL FIELD LAGRANGIAN

In the phase space of the tensor functions \( h_{\mu\nu} \) and their derivatives \( h_{\mu\nu;\sigma} \) the Lagrangian \( L^{\text{grav}} \) is considered as a function of the independent variables \( h_{\mu\nu} \) and \( h_{\mu\nu;\sigma} \) the explicit form of which is determined by solution of self-action equation (5). From this viewpoint self-action equation (5) is a linear differential equation in first-order partial derivatives for the one unknown function \( L^{\text{grav}} = L^{\text{grav}}(h_{\mu\nu}, h_{\mu\nu;\sigma}) \).

*The principle involved in constructing the theory does not depend on choice of one or the other expression for the energy tensor.

†Everywhere below we use the dimensionless gravitational potential utilizing the rule \( \kappa h_{\mu\nu} = h_{\mu\nu} \), although the notation remains as before.