\[ a_1 = \left( \lambda + \lambda_i \left( \frac{1 - \sqrt{1 - 4R_1 R_2}}{2R_1} \right)^2 - B \left( \frac{1 - \sqrt{1 - 4R_1 R_2}}{2R_1} \right) \right) \frac{S_k^c}{S_k}; \]
\[ a_2 = \left[ \lambda R_1 + \lambda_i \left( \frac{1 + \sqrt{1 - 4R_1 R_2}}{2} \right)^2 - B R_1 \left( \frac{1 + \sqrt{1 - 4R_1 R_2}}{2} \right) \right] \frac{S_k^c}{S_k}; \]
\[ w = \frac{2R_1^2 + 2R_2 M^2 - C}{2S_k S_m}. \]

LITERATURE CITED


ASYMPTOTICALLY FINITE SU(N) MODELS OF FIELD THEORY

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Models finite in the one-loop approximation are constructed for quantum field theory. In particular, a finite model with two Higgs multiplets is constructed on the basis of the SU(N) group. Violation of the conditions on the coupling constants induces ultraviolet- or infrared-asymptotic finiteness. The behavior of the effective coupling constants in an external gravitational field is also investigated.

INTRODUCTION

The investigation of renormalization group equations by and large determines the properties of grand unification theories (GUTs) (see, e.g., [1]). This is attributable, for example, to the fact that the requirement of asymptotic freedom with respect to all coupling constants imposes constraints on the gauge group and multiplet composition of the GUT and also on the values of the coupling constants (see, e.g., [2-6]). The necessity of asymptotic freedom is associated, in particular, with the application of perturbation theory, which involves expansion in the coupling constants. A natural alternative to asymptotically free GUT models in this sense is found in finite theories, in which the effective constants have constant (independent of the renormalization parameter) and, at the same time, sufficiently small values [7].

Finite gauge theories have come under intense study over the last few years. It is a well-known fact that a field theory finite in all orders can be constructed for a theory with expanded (N = 8)-supersymmetry [8, 9]. Various aspects of finite theories have been discussed [10-15], including the relationship between finiteness at the one-loop level and in higher perturbation orders. Such a relationship indeed occurs for supersymmetric models (see, e.g., [14]). On the other hand, Böhm and Denner [13] note that finiteness is not directly related to supersymmetry. They have derived a series of nonsupersymmetric models finite at the one-loop level. All these models have a multiplet composition that admits supersymmetry, which is broken only by "incorrect" values of the coupling constants. A theory with arbitrary values of the coupling constants can be treated in this situation. By virtue of the radiation


corrections, such a theory restores the finiteness asymptotically [16] in the infrared and ultraviolet limits, where, as a rule, asymptotic supersymmetry exists in the IR limit [16, 17].

From the standpoint of the physical characteristics of finite theories it would be useful to have a finite GUT model with a phenomenologically justified structure. Such a model can be used, for example, on a gauge group that contains SU(5) as a subgroup and which must also include a sufficiently exhaustive sector of scalar fields. We require that this model have zero β-functions at the one-loop level. As a first step we consider a SU(N)-based massless theory with a fixed set of scalar fields and a nonpredetermined number of spinor multiplets. The problem is to find the values of N, the number of spinors of different species, and the coupling constants corresponding to a finite theory. Violation of the conditions on the coupling constants results in the loss of finiteness. The renormalized β-functions in such a theory clearly have fixed points corresponding to a finite theory. Asymptotic finiteness (in the UV or IR limit or both) should be expected in this case.

From the point of view of applications, specifically in cosmology, it is important to investigate finite GUT models in an external gravitational field. The general renormalization structure in an external gravitational field has been studied in detail (see, e.g., the survey [18]). Multiplicative renormalizability requires the introduction of nonminimal interaction of the scalar fields with gravitation. In this case the β-function for nonminimal parameters contains the factor \((\xi_{ij} - \delta_{ij}/6)\), where \(\xi_{ij} = \delta_{ij}/6\) corresponds to a conformally invariant theory. It is obvious that all the β-functions, including \(\beta_{\xi}\), will be equal to zero for a conformally invariant generalization of a finite model. On the other hand, a theory in an external gravitational field is inevitably accompanied by vacuum divergences, which depend only on the external fields. We shall focus our attention on divergences in the sector of material fields. When the equality \(\xi_{ij} = \delta_{ij}/6\) fails, generally speaking, the theory can be asymptotically conformally invariant (see [18] and the literature cited therein). Bukhbinder et al. [19] have undertaken a study of the asymptotic behavior of the effective parameters of nonminimal coupling, based on the supersymmetric finite models obtained in [13]. In every case \(|\xi(t)| \to \infty\) in the UV limit, i.e., asymptotic conformal invariance occurs only for low interaction energies.

Our finite models have various multiplet compositions and group dimensionalities N. It is therefore possible to construct field-theoretical models that are asymptotically finite and asymptotically conformally invariant in the UV limit.

1. BRIEF DESCRIPTION OF THE MODELS

We consider a field theory based on the gauge group SU(N) and including the scalar fields \(\phi^a\) and \(\psi^a\), spinor Dirac fields \(\psi^a, \bar{\psi}^i\), and \(\chi^i\). The action has the form

\[
S = \int d^4x \left\{ -\frac{1}{4} (G^{a}_{\mu\nu})^2 + \frac{1}{2} (D_a \Phi)^2 (D_b \Phi)_{ab} + \frac{g^{a\nu}(D_a \tau^+) \cdot (D_b \tau^+)_{ij}/2) (D_a \tau^+)_{ij}/2) \psi_{\nu}^a + \frac{1}{2} F_{i} \left( \frac{\lambda}{2} \right) \right. \\
+ i \sum_{k=1}^{m} \sum_{\alpha} \sum_{\beta} \left( \frac{\lambda^{(\alpha)}}{2} \right) \bar{\psi}_{\nu}^a \left( \frac{\lambda^{(\beta)}}{2} \right) T_{(k)}^a \left( \frac{\lambda^{(\beta)}}{2} \right) \chi_{\nu}^i - \frac{1}{8} f_{i} \left( \frac{\lambda^{(\alpha)}}{2} \right) \right\}. 
\]

The fields \(\phi^a\) and \(\psi^a\) are taken in the associated representation, and the fields \(\psi_i, \bar{\psi}^i\), and \(\chi^i\) are taken in the fundamental representation of the gauge group, \(i, j = 1, \ldots, N; a, b = 1, 2, \ldots, N^2 - 1; \lambda^{a}/2\) are SU(N) generators; and \(D_a\) denotes covariant derivatives. The Bose sector of the theory (1) has been used previously to investigate the problem of asymptotic freedom in quantum R^2 gravitation with matter [20, 18].

The expression for the β-function of the gauge constant \(g\) is critical from the point of view of one-loop finiteness:

\[
\beta_g = -\frac{1}{(4\pi)^2} b^2 \eta, 
\]

1095