MODELLING ČERENKOV RADIATION OF RELATIVISTIC PARTICLES IN CRYSTALS

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From the aspects of classical mechanics and electrodynamics, an analysis has been performed of the possible influence of the kind of charged particle trajectory on the Čerenkov-radiation spectrum in a crystal. Results of the analytical computation are compared with the data of a computer experiment. It is shown that the influence of the particle trajectory on the Čerenkov radiation spectrum is insignificant in the optical frequency band. The expected effect is possible when utilizing crystals with a superlattice and by observation of radiation in the x-ray frequency range.

1. INTRODUCTION

As a rule, satisfying the conditions for simultaneous observation of different physical processes results in the appearance of new interesting regularities. From this viewpoint Čerenkov radiation is no exception. The papers [1, 2] in which the mechanism of the interaction of two radiations, Čerenkov and synchrotron, can be the proof of this. Results show that this mechanism results in the origination of synchrotron-Čerenkov radiation that possesses a synergetic feature in comparison with the interference effect of the transition and Čerenkov radiation interaction.

Investigation of the process of particle motion near the crystal axes and planes [3, 4] shows that enrichment of the radiation spectrum in the hard x-ray range is due to the presence of three effects, the Vavilov-Čerenkov effect and the anomalous and complex Doppler effects.

On the basis of a quantum-mechanical analysis [5] for the super-barrier motion of particles in a crystal, the hypothesis is advanced that the process of particle scattering by the atomic planes should result in suppression of Čerenkov radiation.

The purpose of this paper is to determine from the aspects of classical mechanics and electrodynamics in what manner the type of particle trajectory in the crystal influences the Čerenkov radiation spectrum. To do this, analytical computations and a machine experiment were performed for two kinds of particle motion in a crystal, rectilinear motion and motion near the crystallographic plane.

2. THEORY

We introduce the notation by using the terminology accepted in [6]:

\[ I = \int_{-\infty}^{\infty} \varphi(t) \exp \left[ i \omega t - iKr(t) \right] dt, \quad (1) \]

\[ dI_{\omega,\kappa} = \frac{e^{-i}}{4\pi^2n} \left| \left[ \kappa \times I \right] \right|^2 d\omega d\omega, \quad (2) \]

where \( I \) is the path of radiation formation, \( dI_{\omega,\kappa} \) is the radiation spectrum density at the frequency \( \omega \) in the direction \( \kappa \) in the solid angle \( d\omega \); \( n \) is the index of refraction of the medium, \( \kappa \) is the wave vector, \( r(t) \) and \( v(t) \) are the particle radius vector and velocity. Formulas (1) and (2) are valid for the case of infinite particle motion in a medium. If we
limit the time of particle motion in the medium, then two terms appear in (1) and (2) that correspond to finiteness of the trajectory (see [7]), and we omit them yet; they will be taken into account when obtaining the final result (reflection and refraction are also not yet taken into account for the radiation).

We will consider the medium homogeneous, isotropic with refractive index \( n = \text{const} \), the charge motion to occur in the time interval \( (0, T) \), the radiation to be observed strictly on the Čerenkov cone at an angle \( \theta = \theta_c \) relative to the longitudinal particle velocity component, i.e., the following condition is satisfied

\[
I - \beta \parallel n \cdot \cos \theta_c = 0.
\] (3)

In the case of uniform rectilinear motion the integration of (2) with (3) taken into account will give us an expression of the form

\[
dI_o = I_0 \cdot \omega^2 \cdot n \cdot T^2 \cdot \sin^2 \theta d\omega d\phi.
\] (4)

where \( I_0 = \frac{e^2}{4\pi^2 c^2} \) and \( T \) is the total radiation time. When particles enter the crystal at a small angle to the crystalline plane, a specific kind of trajectory occurs (see [8], say). In particular, the particle motion near the crystalline plane is often described by equations of the form

\[
r(t) = v_x t + r_x \cos \omega t; \quad \varphi(t) = \varphi - r_y \omega^2 \cdot \sin \omega t,
\] (5)

where \( |r_x| \) is the amplitude of transverse particle displacement relative to the direction \( \psi \parallel Oz; \); \( \omega \) is the frequency of vibration around the plane. We obtain for the trajectory (5) from the expression (1)

\[
I = I + I_-,\n\]

where

\[
I = \int_0^T \psi \exp \left| i \omega t - ikr(t) \right| dt;
\]

\[
I_- = - \int_0^T r_y \exp \left| i \omega t - ikr \right| dt.
\] (6)

Separating the path the particle traverses into sections that are multiples of the period of the vibrations, the following relationship can be obtained from (6) in the general case

\[
I_+ = \psi \int_0^{\frac{T}{2}} \sin \frac{N}{2} \omega T^* (1 - \frac{3\pi}{2} \cos \theta) \sin \frac{1}{2} \omega T^* (1 - \frac{3\pi}{2} \cos \theta) \left( \sin \omega t \cdot e^{i\omega t (1 - \frac{3\pi}{2} \cos \theta)} \right) dt.
\]

\[
I_- = -r_y \omega \int_0^{\frac{T}{2}} \sin \frac{N}{2} \omega T^* (1 - \frac{3\pi}{2} \cos \theta) \sin \frac{1}{2} \omega T^* (1 - \frac{3\pi}{2} \cos \theta) \sin \omega t \cdot e^{i\omega t (1 - \frac{3\pi}{2} \cos \theta)} \cdot dt.
\] (7)

Taking account of the condition (3), the expressions (7) finally yield the following formulas (on the Čerenkov cone):

\[
I_+ = \frac{2\pi N}{\omega} \psi \cdot J_0 (kr_x); \quad I_- = 0,
\] (8)

where \( N \) is the number of periods of trajectory vibrations, and \( J_0 (kr_x) \) is the zero order Bessel function. The spectral intensity of the radiation on the Čerenkov cone is determined from the trajectory (5) as