TIDAL-FRICTION THEORY OF THE EARTH-MOON SYSTEM

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"Amicus Plato, amicus Socrates, sed magis amica veritas"
Ammonios Saccas (3rd century B.C.)

Abstract. The standard discussion of tidal friction in the Earth–Moon system has been that given by Jeffreys in successive editions of The Earth over the past several decades. It is herein shown to contain several errors vitiating its results. The dynamical equation utilised for finding the rate of change of angular velocity of the Earth fails to take account of the fact that the moment of inertia of the Earth may be changing with time, and all subsequent equations which depend on this are incorrect as a result. Simple equations have been left unsolved that ought to have been solved, and the alleged numerical conclusions in no way follow from the values set down initially for the observed apparent secular accelerations of the Moon and Sun.

The revised dynamical equations are shown to enable the lunar and solar tidal couples to conform to theory, and may imply that the moment of inertia of the Earth is decreasing at a non-negligible rate. Recognition of this is the key to the whole problem. The only available hypothesis providing adequate contraction is that following from the phase-change theory of the nature of the terrestrial core, and the value of the rate of decrease of moment of inertia calculated from this is in close agreement with that implied by modern improved values of the secular accelerations.

1. Introduction

The standard discussion of tidal friction has long been that given by Jeffreys in successive editions of The Earth, of which there must have been some tens of thousands of copies distributed worldwide during the past half-century. It may now come as a surprise to many readers and owners of the volume to learn that the whole discussion is invalidated by serious dynamical as well as numerical errors. The dynamical equations relied upon are incorrect, rendering the consequent treatment fallacious; and the numerical results as claimed rest on alleged adopted values that are entirely at variance with the values set down at the outset for the apparent secular accelerations of the Moon and Sun.

It will assist the reader, in order to understand the nature of the errors, if the theory as presented by Jeffreys is first summarised, regardless of the fact that it will later be shown to be invalid. The data claimed to be used are from analyses of ancient-eclipse records by Fotheringham and others, and are the values of the so-called apparent secular accelerations.

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of the Moon and Sun, denoted by $v$ and $v'$, which are defined as

$$v = \dot{n} - \frac{n}{\omega} \omega = 10''.44 \text{ per century}^2,$$

$$v' = \dot{n}' - \frac{n'}{\omega} \omega = 3''.60 \text{ per century}^2,$$

where $n$ and $n'$ are the mean-motions of the Moon and Sun, respectively, and $\omega$ denotes the angular velocity of the Earth. These numerical values are those that were adopted by de Sitter (based on the studies of Fotheringham), and are subject to standard-errors of about $\pm 10\%$, but as the errors themselves are not in any way relevant to the present exposition they are omitted for the sake of clarity and brevity.

2. The Theory Presented by Jeffreys

As a result of tidal interaction between the Earth and Moon, a couple $N$ acts upon the Moon increasing its orbital angular momentum, and a reaction-couple $-N$ acts upon the Earth decreasing its rotatory momentum. Similarly a couple $N'$ acts on the Sun in its orbit about the Earth, and hence a couple $-N'$ upon the Earth. The masses of the Earth, Moon, and Sun are denoted respectively by $M$, $m$, and $m'$, the mean Earth–Moon distance by $c$, and the mean Earth–Sun distance by $c'$. Since the quantities $n$ and $c$, although changing, always remain related by $n^2c^3 = G(M + m)$, where $G$ is the constant of gravitation, and similarly for $n'$ and $c'$, they can be replaced by single parameters $\xi$ and $\xi'$ by writing, following Jeffreys's notation

$$c = c_0\xi^2, \quad n = n_0\xi^{-3}; \quad c' = c'_0\xi'^2, \quad n' = n'_0\xi'^{-3}.$$

Since the orbital angular momentum of the Earth–Moon system is $Mmc^2n/(M + m)$; and since $c^2n = c_0^2n_0\xi$, the relevant dynamical equations may be written

$$\frac{Mmc^2n_0}{M + m} \frac{d\xi}{dt} = N,$$

$$\frac{m'Mc_0^2n_0}{m' + M} \frac{d\xi'}{dt} = N',$$

the mass of the Moon being negligible in the second equation. These equations, as given by Jeffreys, appear to be unexceptionable dynamically, and are not called in question here. Clearly they enable $\dot{n}$ and $\dot{n}'$ of (1) to be expressed in terms of $N$ and $N'$, but to find $v$ and $v'$, the value of $\dot{\omega}$ is also required analytically, and with the moment of inertia of the Earth denoted by $C$, Jeffreys writes for this purpose

$$C \frac{d\omega}{dt} = -N - N'.$$

Since the value of $\dot{n}'$ may readily be shown to be negligible compared with terms arising alongside it, Equations (3), (4), and (5) inserted in (1) lead to