COLLISIONAL GROWTH OF PLANETESIMALS

RICHARD GREENBERG
Planetary Science Institute, Tucson, Arizona, U.S.A.

(Received 24 August, 1979)

Abstract. Safronov's (1972) demonstration that relative velocities of planetesimals would be comparable to the dominant size bodies' escape velocities, combined with a plausible size distribution that has most mass in the largest bodies, yielded his evolution model with limited growth of the largest planetesimal with respect to its next largest neighbors. A numerical simulation of planetesimal accretion (Greenberg et al., 1978) suggests that at least over one stage of collisional accretion, velocities were much lower than the escape velocity of the largest bodies, because the bulk of the mass still resided in km-scale bodies. The low velocities at this early stage may conceivably have permitted early runaway growth, which, in turn, would have kept the velocities low and permitted continued runaway growth of the largest bodies.

The planetesimal hypothesis involves growth of full size planets from grains of material condensed from the early nebula. Work by Safronov (1969) and Goldreich and Ward (1973) suggested that the earliest stage of accumulation was due to gravitational instability in a thin dust layer which yielded km-scale planetesimals, although some accretion may have occurred as the dust settled towards a plane (Weidenschilling, 1979). Models of subsequent growth by collisional evolution depend on relative velocities amongst planetesimals. The velocities govern the probability of collisions, the outcomes of impact (erosion and fragmentation vs. accretion) and, by determining orbital eccentricities, the extent of the feeding zones of growing planetesimals.

Safronov (1969) showed that relative velocities can be expected to be on the order of the escape velocity of the dominant bodies, assuming an equilibrium between collisional damping and gravitational stirring. In a collisionally relaxed system, one might expect the size distribution to obey a power law with most mass in the larger bodies.* In that case, the mean random relative velocity is comparable to the escape velocity of the largest body in a swarm. As a result, even the largest body’s gravitational cross-section can never get much larger than its geometrical size; runaway growth of the largest body is thus limited. For example, Safronov (1969) concluded that the second largest body in the Earth’s zone would have about one-tenth the radius of the Earth. It has been emphasized (cf. Greenberg, 1979) that this solution represented a lower limit to the size of the second largest body.

However, Levin (1978) suggested a way in which runaway growth may have been

*Such a distribution is suggested by the theory of Dohnanyi (1969). However, Davis et al. (1979) note that a different outcome may result if the gravitational self-binding of colliding bodies is taken into account.

Paper presented at the European Workshop on Planetary Sciences, organised by the Laboratorio di Astrofisica Spaziale di Frascati, and held between April 23-27, 1979, at the Accademia Nazionale del Lincei in Rome, Italy.

unleashed. He noted that according to Safronov once one body becomes significantly larger than any other, relative velocities begin to be on the order of the escape velocities of the largest bodies on the continuous portion of the size distribution. Once relative velocities become much smaller than the largest body's escape velocity, its gravitational cross-section increases strongly with size and rapid run-away growth is possible, according to Levin.

But could any one body ever get so large compared to its neighbors that control of velocities is transferred to smaller ones? Safronov (1979) argues persuasively that it could not. He points out that as the largest body grows, the corresponding increase in relative velocities broadens the extent of its feeding zone. The expanding feeding zone will begin to merge with those of other large bodies. Hence, even if one body grows much larger than others in its zone, other large bodies are continually being added to the zone. The largest body is always accompanied by others of comparable mass, thus prohibiting the Levin scenario.

This argument hinges on the requirement that the largest bodies in adjacent independent feeding zones are of comparable size immediately before the feeding zones merge. The growth rate of the largest embryo in each zone is given by

$$\frac{dm}{dt} = \pi r^2 \rho_c v \left(1 + \frac{v_e^2}{v^2}\right)$$

where the quantity in parenthesis is nearly constant as discussed above and $\rho_c$ (the density of the cloud of smaller bodies) is inversely proportional to the cloud thickness, which in turn goes as $v$. Thus $\frac{dm}{dt} \propto r^2$. This implies that the mass ratio of largest bodies in independent zones tends toward unity.

The coupling of the relative velocity to the size of the largest body in a zone is crucial to Safronov's elegant model. However, numerical studies (Greenberg et al., 1978) of coupled collisional evolution of velocity and mass distributions indicate that over a crucial stage in the growth of planetary embryos the mass distribution may not have been the shallow-sloped power law assumed by Safronov. Our model incorporates realistic treatment of the wide range of collisional outcomes possible for various sizes and impact velocities. We start with a swarm of all km-sized planetesimals, i.e., the distribution of bodies larger than 1 km in diameter is initially very steep. As time goes on, the distribution of larger bodies becomes less steep as more and more large bodies are produced by accretion. Nevertheless, by the time ($\sim 2 \times 10^4$ yr) a few 500 km bodies ($\sim 1/AU^2$) are produced, the distribution is still so steep that most of the mass of the system still resides in planetesimals only a few km in diameter. At that point, the random relative velocities of all bodies are on the order of the escape velocities of the km-scale bodies. This result supports Safronov's analytic work on relative velocities in that these small planetesimals are the dominant component of the population. But the result indicates that the size distribution can be much steeper than has been generally assumed. Hence, it contradicts Safronov's assumption, crucial to his arguments above, that the system is always in equilibrium with most of the mass in the largest bodies. At least over this