Geostatistical Estimation of Multi-Phase Structures

Amilcar Soares

A method is proposed for the characterization of the disjoint shapes of a multi-phase set. The method uses a global structural function and provides estimates of the complete mosaic of phases, honoring the individual volume proportions inferred from the experimental samples. The estimates of shapes can be improved by local conditioning to the covariance of each phase and to geometrical characteristics such as spatial orientation of the different strata. The mapping of uncertainty zones for individual phases is one advantage of using a geostatistical approach to characterize the morphology of qualitative (non-numerical) variables.

KEY WORDS: multi-phase sets, geometric modeling, morphological kriging.

INTRODUCTION

Examples of multi-phase sets where the components have disjoint shapes and there are structural relations between the phases are: a distribution of geological facies within a region, a mosaic of different types of mineralization, and the spatial distribution of a set of classes of a non-additive variable. The separate, geostatistical estimation of the shape of each phase or component by, for example, kriging an indicator variable, has two major drawbacks in these applications fields characterized by a set of phases without any order relation: (1) it calls for each individual phase covariance model, which are unavailable in most situations, and (2) moreover, the separate estimation of different phases does not honor the global architecture of the entire set and consequently can lead to some global inconsistencies. For example, the estimated relative volumes may not sum to 1 (see Solow, 1986), producing significant zones where the points are estimated to belong to more than one phase. Some authors face this problem by coupling geostatistics with deterministic geometric modeling (Dowd, 1986; Mallet, 1989).

The method proposed in this paper provides estimates of the probability of a point to belong to any particular phase; the estimation is based on a global

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2CVRM, Technical University of Lisbon, IST, Av. Rovisco Pais, 1096 Lisboa Codex, Portugal.
structural analysis of the multi-phase set that is quantified by a single covariance of the set-probability of two points belonging to the same phase. The estimated probabilities are classified in order to produce a set of disjoint phases with the same relative volumes inferred by the experimental samples. When significant differences between the global and local structural parameters are observed, the estimation process can also be conditioned to some local morphological features, e.g., spatial orientation of the strata, and to the covariances of each individual phase, when they are available. The mapping of estimated uncertainty zones (areas within which there is a low probability of belonging to one or all phases) is a consequent advantage of the use of the geostatistical approach to the problem. A simple case study on a two-dimensional geological map is used to illustrate the method.

**ESTIMATION OF MULTI-PHASE STRUCTURES**

Denote the probability that a point \( x \) belongs to a phase \( i \) by \( p_i(x) \):

\[
p_i(x) = \text{prob} \{ x \in \text{phase } i \}, \quad i = 1, N
\]

The objective of the proposed method is to obtain, on a grid of points, the estimated values \( p_i^*(x) \), \( i = 1, N \), such that:

\[
\sum_{i=1}^{N} p_i^*(x) = 1, \quad \text{for all } x \tag{1}
\]

The estimation \( p_i^*(x) \) is obtained by taking a linear combination of the experimental indicator data. At each sample location \( x_\alpha \), a vector of indicator values \( K_1(x_\alpha), K_2(x_\alpha), \ldots K_N(x_\alpha) \) can be defined where:

\[
K_i(x_\alpha) = \begin{cases} 1 & \text{if } x_\alpha \in i \\ 0 & \text{if } x_\alpha \in j \end{cases} \quad i \neq j \tag{2}
\]

The general form of the linear estimator of the probability of a point \( x \) to belong to the phase \( i \), is:

\[
p_i^*(x) = K_i^*(x) = \sum_{\alpha} \lambda_{i\alpha} K_i(x_\alpha)
\]

where the weights \( \lambda_{i\alpha} \) depend on the neighborhood sample location \( x_\alpha \) and the phase \( i \). To verify Eq. (1), \( \sum p_i^*(x) = 1 \), it is sufficient that the weights \( \lambda \) are the same for all phases \( i \) \( (\lambda_{\alpha 1} = \lambda_{\alpha 2} = \ldots = \lambda_{\alpha N} = \lambda_\alpha) \) and to impose the unbiasedness condition \( \Sigma \lambda_\alpha = 1 \) (see Journel, 1983). If a geostatistical approach is used to obtain the new estimator:

\[
K_i^*(x) = \sum_{\alpha} \lambda_\alpha K_i(x_\alpha) \quad i = 1, N \tag{3}
\]