FORMATION OF PLANETARY SYSTEMS:
A SIMPLE SYNTHETICAL APPROACH

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Abstract. It is shown that by means of elementary assumptions it is possible to obtain models of planetary systems not too different from the observed case. Such assumptions are shown to be the bulk of more detailed models as those by Dole (1970) and Isaacman and Sagan (1977).

An accurate fit with the features of the Solar System can be obtained only by ad hoc assumptions on various physical quantities and depends also on the possibility of taking into account stochastic processes in the numerical code.

We discuss various cases and attempt to derive best fit values for the free parameters of the model. The hypothesis of a moderate mass depletion in the region of outer planets is strongly supported by our results.

1. Introduction

One of the most striking approaches to the problem of the formation of the Solar System is to derive a general model of planetary accumulation by the a priori assumption of a primaeval rotating nebula in which condensation nuclei (planetesimals) are formed.

In particular this approach is analyzed in the papers by Dole (1970) and more recently by Isaacman and Sagan (1977) and Vityazev et al. (1978).

In this paper we want to point out that few elementary assumptions can be recognized to be the basis of models such as those of Dole (1970) and Isaacman and Sagan (1977). By means of these assumptions alone we can obtain realistic synthetical planetary systems and fit the Titius–Bode law, its deviations due to the planetary masses (Farinella and Paolicchi, 1977) and the relation of the mean planetary spacing with the mean planetary mass (Colombo, 1975).

We also show that, in order to avoid ad hoc hypotheses on the initial physical conditions and on the accumulation process, it is necessary to assume mass depletion in the outer regions. Moreover, eccentricity effects and those connected to stochastic processes (fluctuations, collisions and so on) should probably be taken into account.

2. Basic Assumptions

We start with an axially symmetrical rotating disk of dust and gas, with a surface density $\sigma(r)$, and a ratio $\sigma_{\text{gas}}/\sigma_{\text{dust}} = 50$, near to solar values.
Let us assume that dust initially, and planets after the process of formation, maintain circular orbits. Let us also assume the following.

(1) The mechanism of planets formation is the same along the system.
(2) All the dust is captured by the planets.
(3) Every planet captures matter in a ring defined by

\[ x/r = \theta(m/M)^\beta, \]  

where \( x \) is the ring width, \( r \) the distance of the planet from the Sun determined by angular momentum conservation, \( m \) and \( M \) are the planet and the Sun masses, \( \theta \) and \( \beta \) are two free parameters (Dole assumes \( \theta = 1, \beta = \frac{1}{2} \)).

(4) The gas capture can be considered in three ways.
(a) Planets capture only the dust.
(b) Planets capture all the dust and the gas.
(c) Planets capture all the dust and part of the gas.
Cases (a) and (b) can be considered equivalent, formally varying the density by a factor 51.

3. One Component Model

Assuming that the disc mass is much lower than that of the Sun, the velocities are nearly Keplerian; so that we have

\[ m_i = 2\pi \int_{R_i}^{R_{i+1}} \sigma(r) r \, dr, \]  

\[ r_i = \left[ \frac{\int_{R_i}^{R_{i+1}} \sigma(r) r^{3/2} \, dr}{\int_{R_i}^{R_{i+1}} \sigma(r) \, dr} \right]^2, \]  

where \( R_i \) and \( R_{i+1} \) are the boundaries of the ring. If we define

\[ I_\sigma(i, j, h) = \int_{R_i}^{R_j} \sigma(r) r^h \, dr, \]  

Equation (1) becomes

\[ (R_{i+1} - R_i) \left[ \frac{I_\sigma(i, i + 1, 1)}{I_\sigma(i, i + 1, \frac{3}{2})} \right]^2 = \theta [(2\pi/M) I_\sigma(i, i + 1, 1)]^\beta. \]  

If \( R_k = \alpha_k R \) (\( \alpha_k \ll 1, R \) is the nebular radius) we have

\[ I_\sigma(i, i + 1, h) = R^{h+1} \int_{\alpha_i}^{\alpha_{i+1}} \sigma(\alpha) \alpha^h \, d\alpha \]

\[ = R^{h+1} \mathcal{F}(i, i + 1, h), \]  

where \( \mathcal{F} \) is the function that captures the dependence on the nebular radius.