Using the smallness of the deformation parameter of the nucleus, we obtain simple explicit expressions for the form factors of electroexcitation of the low-lying rotation-vibration states of light, deformable, even-even nuclei. The expressions satisfactorily describe the experimental data on the excitation of collective nuclear states by the inelastic scattering of fast electrons.

In the theoretical study of the process of electroexcitation of low-lying collective states of light, deformable, even-even nuclei, the calculation of the cross section reduces to the numerical evaluation of multiple integrals [1], which in the final analysis makes this type of analysis difficult to carry out. However, the deformation parameter of the nucleus \( \beta_0 \) is fairly small for all nuclei [2, 3]. Although the wave functions \( \varphi_{n\ell\gamma}(\beta) \) describing the \( \beta \)-oscillations of the nucleus cannot themselves be expanded in a power series in \( \beta_0 \), the inelastic and elastic electron form factors are sufficiently smooth functions of this parameter and therefore can be expanded. It turns out that in each term of the power series in \( \beta_0 \) all integrals with respect to the variable \( \beta \) and also with respect to the Euler angles can be evaluated, at least for the wave functions \( \varphi_{n\ell\gamma}(\beta) \) of [4]. In addition, as shown by numerical estimates, even the first two or three terms of the series give results that are sufficiently close to the exact values of the form factors, even for a maximum value of the deformation parameter \( \beta_0 \approx 0.5 \), and thus the form factors are obtained in explicit form.

The cross section for electroexcitation of a light nucleus (the transition \( nI\tau \rightarrow n'I\tau' \)) can be written as a product of the Mott cross section and the square of the modulus of the form factor \( F_{nI\tau}^{n'I\tau'} \), which, for a fixed value of the nonaxiality parameter \( \gamma \), takes the form [1]:

\[
|F_{nI\tau}^{n'I\tau'}|^2 = \frac{1}{2I + 1} \sum_{\ell M' M} \int d\Omega d^2 \mu_{I'M'M'}(\theta, \gamma) \varphi_{n\ell\gamma}(\beta_0) \varphi_{n'I\gamma'}(\beta_0) F(q) \mu_{M \ell M'}(\theta, \gamma) \varphi_{n\ell\gamma}(\beta_0) \left| \frac{d}{d\Omega} \right|^2,
\]

where

\[
F(q) = \frac{1}{d} \int dr e^{i \mathbf{q} \cdot \mathbf{r}} 
\]

Assuming that the surfaces of equal charge density of the nucleus \( \rho(r) \) can be represented as a set of similar ellipsoids with three unequal axes, we can use a similarity transformation of the coordinates \( r' = r/a_i \) to the intrinsic system of the nucleus to transform to a spherically symmetric charge density distribution \( \rho_0(r') \), and if the condition \( a_1a_2a_3 = 1 \) holds the ellipsoid transforms into a sphere. The charge form factor will now take the form [1]:

\[
F(q) = F_0(q') = \frac{4\pi}{6} \int d\Omega' j_0(q' r') \rho_0(r'),
\]

where \( \theta \) and \( \psi \) are two of the Euler angles, while the third Euler angle in (1) can be integrated out.

Since the functions \( \varphi_{nI\tau}(\beta) \) have sharp maxima for values of \( \beta \) close to \( \beta_c \), we can substitute in (1) an expansion of \( F(q) \), or \( F_0(q') \), in powers of \( \beta \), where \( \beta_0 \) is small:
and keep only the first several terms of the series.

Using the functions $\Phi_{n\ell\tau}(\beta)$ from [4], which fall off for large $\beta$ as Gaussians, we obtain the following explicit expressions for the form factor of elastic scattering of electrons $F_\ell^0(q)$ and the form factors of elastic scattering accompanied by the excitation of the first few low-lying collective nuclear levels:

$$F_\ell^0(q) = F_0 + \frac{\gamma}{3} F_0 \left( \frac{\partial q}{\partial \beta} \right)_{\beta=0} + \frac{1}{2} \frac{\gamma}{3} \left[ F_0 \left( \frac{\partial q^2}{\partial \beta^2} \right)_{\beta=0} + \frac{\partial^2 F_0}{\partial q^2} \right]_{q=0} - \ldots. $$

$$F_\ell = F_0(q), \quad F'_0 \equiv \frac{\partial F_0}{\partial q}, \quad F''_0 \equiv \frac{\partial^2 F_0}{\partial q^2}. $$

Using the functions $\Phi_{n\ell\tau}(\beta)$ from [4], we obtain the following explicit expressions for the form factor of elastic scattering of electrons $F_\ell^0(q)$ and the form factors of elastic scattering accompanied by the excitation of the first few low-lying collective nuclear levels:

$$F_\ell^0(q) = F_0 + \left( \frac{3}{2} \mu^2 \right)_{2\pi} (2s_0 + 1) q (2qF_0 - F'_0), $$

$$F_{001}^{\ell} = F_0 + \frac{3}{2} \mu^2 \left[ G_1 \Gamma \left( \frac{s_{2\tau} + s_{01} + 1}{2} \right) + \frac{3}{2} \mu G_2 \Gamma \left( \frac{s_{2\tau} + s_{01} + 3}{2} \right) \right] \left[ \Gamma \left( s_{01} + \frac{1}{2} \right) \Gamma \left( s_{2\tau} + \frac{1}{2} \right) \right]^{-1/2}, $$

$$F_{001}^{\ell} = \frac{3}{2} \mu^2 (16 \sqrt{2} \pi) (2s_0 + 1) q (2qF_0 - F'_0), $$

$$F_{001}^{\ell} = \frac{3}{2} \mu^2 \left[ G_1 \Gamma \left( \frac{s_{2\tau} + s_{01} + 1}{2} \right) + \frac{3}{2} \mu G_2 \Gamma \left( \frac{s_{2\tau} + s_{01} + 3}{2} \right) \right] \left[ \Gamma \left( s_{01} + \frac{1}{2} \right) \Gamma \left( s_{2\tau} + \frac{3}{2} \right) \right]^{-1/2}, $$

$$F_{001}^{\ell} = \frac{1}{2} \frac{3}{2} \mu^2 \left[ G_1 \Gamma \left( \frac{s_{2\tau} + s_{01} + 1}{2} \right) + \frac{3}{2} \mu G_2 \Gamma \left( \frac{s_{2\tau} + s_{01} + 3}{2} \right) \right] \left[ \Gamma \left( s_{01} + \frac{1}{2} \right) \Gamma \left( s_{2\tau} + \frac{3}{2} \right) \right]^{-1/2}. $$

Here $s_{2\tau} = \frac{1}{2} (1 + \sqrt{1 + 2s_0 + 3/\mu^2})$.

$$G_1 = \frac{1}{4} \sqrt{\frac{5}{2}} qF_0 \left[ A_{20}(\gamma) \cos \gamma + A_{20}^*(\gamma) \sin \gamma \right], $$

$$G_2 = \left( 3q (2\pi) \right) (4qF_0 + 3F_0') \left[ A_{20}(\gamma) \cos 2\gamma - A_{20}^*(\gamma) \sin 2\gamma \right], $$

$$G_3 = \left( q (2\pi) \right) \left( qF_0 - F'_0 \right) \left[ A_{40}^* (\gamma) (1 + 5 \cos^2 \gamma) + V^{1/3} A_{40} (\gamma) \sin 2\gamma + V^{1/3} A_{40}^* (\gamma) \sin 2\gamma \right], $$

and the rest of the notation is conventional, and the same as used in [1-4]. Explicit expressions for the charge form factors $F_\ell(q)$ corresponding to different charge density distributions in the nucleus have been given in [1]. We note that the softness parameter $\mu$ of the nucleus is defined as in [4]; it is equal to the similar parameter $\mu'$ in [2] multiplied by $\sqrt{2}$.

Unlike the approach of [1], in our approach it is possible to give a straightforward calculation of the electron form factors of deformable ($\mu \neq 0$) nuclei not only for simple one-parameter charge density distributions such as a uniform distribution and a Gaussian distribution, but also for the two-parameter Fermi distribution. The use of this distribution leads to a good description of the behavior of the experimental form factors as functions of the momentum transfer $q$, near both the first and second maximum. We note that in general the Gaussian distribution does not lead to a second maximum, while the uniform distribution significantly overestimates the form factor near the second maximum, when compared to the experimental value.

For illustration, we show the results for the form factors of electroexcitation in $^{26}$Ne, $^{24}$Mg, and $^{28}$Si with $n' = 0$, calculated from (6) and (7) (Figs. 1 through 4). The square of the modulus of the form factor is shown as a function of the product of $q$ and the radius of an equivalent uniform spherically symmetric charge distribution [1]. Two sets of values of the softness parameter $\mu' = \mu / \sqrt{2}$ were used for the three different nuclei: $\mu'_1 = 0.2, 0.4$, and 0.4 [1] and $\mu'_2 = 0.4, 0.5$, and 0.15 [5]. The solid and dash-dotted curves