QUANTIZATION OF PARTICLE SYSTEM WITH ANOMALOUS MOMENTS

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A procedure is outlined for Hamiltonization of a particle system with anomalous magnetic moments and presumed electrical dipole moments, in the "zero-plane" formalism. Hamiltonization is undertaken in the covariant Lorentz gauge $\delta_\mu A^\mu = 0$.

Previously, the Hamiltonization of a system of interacting fermion and boson fields was considered in the "zero-plane" formalism, taking account of the anomalous magnetic and hypothetical dipole moment of the fermions. The Hamiltonization was undertaken in the gauge $A^0 = A_3 = 0$, which is an analog of the Coulomb gauge in the Cartesian coordinate system.

In the present work the same system is Hamiltonized but in a covariant Lorentzian gauge $\delta_\mu A^\mu = 0$.

Taking the Lorentzian gauge into account allows the action function of the free magnetic field to be rewritten in the form

$$-\frac{1}{4} \int F_{\mu\nu} F^{\mu\nu} d\Omega = -\frac{1}{2} \int \partial_\mu A_\nu \partial_\nu A^\mu d\Omega.$$

The density of the Lagrangian of the whole system here takes the form

$$L = \frac{i}{2} (\bar{\psi} \gamma^\rho \partial_\rho \psi - \partial_\rho \bar{\psi} \gamma^\rho \psi) - \bar{\psi} (\gamma^\rho A_\rho - m) \gamma^\nu \gamma^\mu \partial_\nu A_\mu + m \gamma^\nu \gamma^\mu \partial_\nu A_\mu. \quad (1)$$

The equations of motion of the dynamic variables resulting from Eq. (1) are

$$\{\gamma^\nu (\partial_\nu - eA_\nu) + m \gamma^\nu \partial_\nu A_\mu - m \} \gamma^\nu \partial_\nu A_\mu = 0; \quad (2)$$

$$\{\partial_\nu + eA_\nu \} \bar{\psi} \gamma^\nu \gamma^\mu \partial_\nu A_\mu - m \bar{\psi} (\gamma^\rho A_\rho - m) = 0; \quad (3)$$

$$\partial_\mu \partial^\mu A_\nu = e\bar{\psi} \gamma^\rho \left\{ \frac{\partial_\nu}{\mu} \bar{\psi} (\gamma^\rho A_\rho - m) \right\}. \quad (4)$$

Equations (2) and (3) formally coincide with Eqs. (3) and (4) of [1], but this coincidence is only formal: the present equations include four independent dynamic variables $A_\mu$, whereas the corresponding equations in [1] include only two: $A_1$ and $A_2$.

Conversion to "zero-plane" formalism is by means of the procedure already described in [1]: by projecting Eqs. (2) and (3) onto two orthogonal subspaces using the operators $P^0_+$ and $P^0_-$. The result of this projection onto the $(+)$ and $(-)$ subspaces of Eq. (2) is, respectively,

$$[P_+ \pm i\gamma^\rho F_{\rho\lambda}] \gamma^\mu \gamma^\sigma \gamma^\nu \gamma^\rho \gamma^\omega] \gamma^\nu \gamma^\rho \gamma^\mu = \frac{1}{2} \left\{ \gamma^\mu \gamma^\sigma \gamma^\nu \gamma^\rho \gamma^\omega \right\} \gamma^\nu \gamma^\rho \gamma^\mu.$$

*The notation of [1] is used here.


Conversion to the zero-plane variables and eliminating the variables \(\psi^+\) and \(\psi^-\) gives the result (as before, Latin indices take the values 1, 2 and Greek indices the values 0-3). Analogous equations may be obtained for the functions \(\psi^+\) and \(\psi^-\) projecting Eq. (3). Equation (6) shows that \(\psi^+\) may be expressed in terms of \(\psi^-\) taken at the same "instant of time \(u^2\)" as \(\psi^+\)

\[
\psi^+ = \frac{1}{2} \gamma^\mu D^\mu [\gamma^\alpha F_{\alpha\mu} - m - i\gamma^\alpha (b^\mu F_{\alpha\mu} - b^\mu F_{\mu\alpha})] \psi^-.
\]

(6')

and hence is not an independent dynamic variable. An analogous statement applies to \(\psi^+\).

Conversion to the zero-plane variables \(\hat{\psi}^+\), \(\hat{\psi}^-\), \(\hat{\psi}^\mu\) in the Lagrangian in Eq. (1) and eliminating the variables \(\psi^+\) and \(\psi^-\) using Eq. (6') and the analogous equation for \(\psi^+\) gives the result

\[
L = \frac{i}{\sqrt{2}} (\hat{\psi}^+ - \partial_0 \hat{\psi}^-) + \sqrt{2} \hat{\psi}^+ \left[ \frac{1}{2} b^0 F_{0\alpha} - e A_0 \frac{1}{2} \gamma^\alpha P_j \right.
\]

\[
+ m - i\gamma^\alpha \left[ b^0 (2\partial_0 A_\alpha - \partial_\alpha A_o) + b^\alpha F_{1\alpha} \right] D^{-1} \left[ - \gamma^\alpha P_j \right.
\]

\[
+ \gamma^\alpha \left[ b^0 (2\partial_0 A_\alpha - \partial_\alpha A_o) - b^\alpha F_{1\alpha} \right] \right] \frac{1}{2} \partial_\alpha A_\beta \partial^\beta A^e.
\]

(7)

The relation \(\partial_0 A_3 = -\partial_3 A_0 + \partial_3 A^e\) is used in deriving Eq. (7); this is a consequence of the gauge.

The momenta conjugate with the dynamic variables \(\psi^\mu\), \(\psi^\mu\), \(A^0\), \(A^1\), \(A^3\) are, correspondingly

\[
\pi^\mu = \frac{i}{\sqrt{2}} \psi^\mu - \frac{1}{\sqrt{2}} \psi^\mu, \quad \pi^0 = -\partial_0 A^0, \quad \pi^3 = -\partial_3 A^3.
\]

As in the case of a Coulomb gauge, the momenta do not include time derivatives of the dynamic variables, and hence in this case the generalized Hamilton–Dirac formalism must be used for Hamiltonization of the system.

The complete system consists of twelve relations, ten of which are the same as in the case of a Coulomb gauge, while the two others take the form

\[
f^5 = \pi^0 + \partial_0 A^0, \quad f^6 = \pi^3 + \partial_3 A^3.
\]

(9)

Poisson brackets between the additional relations and those present in the case of a Coulomb gauge are zero, and hence the relations \(f^5\) and \(f^6\) cannot influence the form of the Dirac brackets between the dynamic variables \(\psi^\mu\); \(\psi^\mu\), \(A_0\) \(A_1\) and \(A^3\) and they are of the same form as in [1].

The nonzero Dirac brackets, in which \(A^0\) and \(A^3\) appear, are

\[
[A^0 (u^0, u), A^3 (u^0, u')] = \frac{1}{2} \text{sign} (u^3 - u'^3) \delta (u - u'); \quad u = (u^0, u^2).
\]

(10)

The Hamiltonian of the system of fields takes the form

\[
H = \int H(u) \, du; \quad H = \frac{1}{2} \partial_\alpha A_\beta \partial^\beta A^e + \sqrt{2} \hat{\psi}^+ \gamma^\mu \psi^- V \hat{\psi}^\mu,
\]

where

\[
V = \sqrt{2} i \pi^0 \partial_0 A_0 + e A_0 + \frac{i}{2} \gamma^\alpha [b^\alpha (2\partial_\alpha A_0 - \partial_c A_0) + b^\alpha F_{1\alpha}]
\]

\[
+ \sqrt{2} \gamma^\alpha [b^\alpha + \partial_\alpha A_0 - b^\alpha F_{1\alpha}] D^{-1} \gamma^\beta \gamma^\alpha [b^\beta (2\partial_\beta A_0 - \partial_c A_0) + b^\beta F_{1\beta})].
\]

(11)