FOCUSING PROPERTIES OF THE ELECTRIC FIELD BETWEEN CHARGED CONICAL SURFACES


The focusing properties of the electrostatic field between two charged conical surfaces are considered. An expression is obtained for the electric field intensity in the capacitor formed by two concentric circular cones with parallel generatrices. The equations of motion of an electron in such a capacitor are solved numerically, and it is shown that first-order focusing can be accomplished in it under specified conditions.

The method of electron spectroscopy is presently applied widely in a number of areas of science and technology [1-5]. The main element of the electron spectrometer is the analyzer in which the electrons are separated by energy. As a number of authors have shown, analyzers based on the focusing properties of the electric field between the plates of a plane capacitor, spherical capacitor, and cylindrical capacitor can be used to analyze the energy spectrum of electrons [4-8]. It was found in a number of investigations that the fields between toroidal, hyperbolic, and elliptical surfaces have good focusing properties [9-11]. Analyzers based on the use of the focusing properties of one of these fields make possible the energy analysis of electrons escaping from a sample at a specific angle. A significantly greater volume of information about a substance being investigated can be obtained by carrying out the energy analysis of electrons escaping from the sample at different angles [12-14]. Energy and angle analysis are usually carried out with the help of one of the above-mentioned analyzers moving with respect to the sample [15].

The focusing properties of the electric field between charged conical surfaces are considered in this work, and it is shown that an analyzer based on the use of these focusing properties can be used to investigate the distribution of electrons both in energy and in escape angle, with the analyzer remaining fixed for all angles being analyzed.

FIELD INTENSITY BETWEEN THE ELECTRODES OF A CONICAL ANALYZER

The flux vector of the electrostatic induction \( \mathbf{E} \) through the surface element \( dS_0 \) on the inner cone (Fig. 1) and through the area \( dS_1 \) located at a distance \( \xi \) from the area \( dS_0 \) are, respectively,

\[
d\Phi_0 = z_0 E_0 \cdot dS_0 \cdot \cos (E_0 \cdot dS_0), \quad d\Phi_1 = z_0 E_1 \cdot dS_1 \cdot \cos (E_1 \cdot dS_1).
\]

The areas of these surface elements are

\[
dS_0 = r_0 \cdot d\zeta \cdot dp \quad \text{and} \quad dS_1 = (r_0 + \xi \cdot \cos \zeta) \cdot d\zeta \cdot dp.
\]

By equating \( d\Phi_0 \) and \( d\Phi_1 \) we obtain:

\[
E_0 \cdot r_0 = E_1 \cdot (r_0 + \xi \cdot \cos \zeta).
\]
Fig. 1. Relative position of the elements of the conical surfaces. The position of the rectangular coordinate system is shown at the lower right.

$$E_i = \frac{E_0 \cdot r_0}{r_0 + \xi \cdot \cos \alpha}.$$  \hspace{1cm} (4)

Let us express $E_0$ in terms of the potential $V$ between the electrodes of the conical analyzer. Since $V = \int_0^d E_i \cdot d\xi$, where $d$ is the distance between the electrodes of the analyzer, we have:

$$V = \int_0^d \frac{E_0 \cdot r_0 \cdot d\xi}{r_0 + \xi \cdot \cos \alpha} = E_0 \cdot r_0 \cdot \ln \left( \frac{r_0 + d \cdot \cos \alpha}{r_0} \right).$$  \hspace{1cm} (5)

Hence

$$E_0 = \frac{V \cdot \cos \alpha}{r_0 \cdot \ln \left( 1 + \frac{d \cdot \cos \alpha}{r_0} \right)}.$$  \hspace{1cm} (6)

and

$$E_i = \frac{V \cdot \cos \alpha}{\ln \left( 1 + \frac{d \cdot \cos \alpha}{r_0} \right) \left( r_0 + \xi \cdot \cos \alpha \right)}.$$  \hspace{1cm} (7)

Taking into account that $r_0 = \rho \sin \alpha$, we obtain:

$$E_i = \frac{V}{\ln \left( 1 + \frac{d \cdot \rho \tan \alpha}{r_0} \right) \left( \rho \cdot \tan \alpha + \xi \right)}.$$  \hspace{1cm} (8)

Thus, the electric field intensity between the plates of the conical analyzer depends only on the $\rho$ distance from the vertex of the cone and on the distance $\xi$ to one of the cones. This result is valid for a sufficiently large ratio $\rho/d$. It was implicitly assumed in the derivation that the electric field intensity $E$ is perpendicular to the areas $dS_0$ and $dS_i$, whereas the very dependence of the intensity $E$ on $\rho$ indicates that it cannot be strictly perpendicular for the area selected in the indicated manner. Elimination of this contradiction is related to second-order corrections which are smaller the farther the considered section of the interelectrode gap is located from the vertex of the cone.

**EQUATION OF MOTION OF THE ELECTRON**

In view of the axial symmetry of the cone, it is sufficient to consider the motion of the electron between the electrodes in the plane passing through the axis of the cone. To write the equations of motion of the electron it is convenient to transfer from the variables $\rho$ and $\xi$ to others, usually denoted $x = \rho$ and $y = \xi$, which is possible in view of the mutual perpendicularity of $\rho$ and $\xi$ (Fig. 1). We will assume that the entrance slit is sufficiently far from the vertex of the inner cone, which is equivalent to the condition $\rho/d \gg 1$ for all points of the trajectory of the electron motion. It is assumed with this notation that the origin of the coordinates is at the vertex of the small cone and the $X$ axis is directed along