A complete solution is given for the problem of impurity excitation by a two-photon field, as a result of which the remaining free quantum passes from one polariton branch of the spectrum to another without change in the wave vector. The decisive role of relaxation constants in the formation of the process is noted. The criteria of independent behavior of photons in this reaction are given. The given process belongs to the class of processes due to disrupted interference of the matrix elements.

The dependence of the quantum energy on the wave number in a medium of two-level atoms is shown in Fig. 1. The possibility of combinational absorption of the two-photon field by impurity, accompanied by transition of the nonabsorbed quantum from one energy branch to another without change in wave vector, was examined in [1, 2]. The intensity of this process was calculated by perturbation theory. Since the photon in the medium interacts with the surrounding atoms before, during, and after the interaction of its partner with impurity, calculation of the intensity of the process requires summation of an infinite sequence of Feynmann diagrams. In this sense, it is necessary to pass beyond the bounds of perturbation theory. Such summation is undertaken in the present work, and thereby the investigation of the process is absolutely completed. At the same time, it is shown that relaxational constants play a determining role in its formation. The physical assumptions, notation, and method of investigation outlined in [2] are adopted here. The wave function describing the interacting two-photon field and the impurity against the background of the medium is sought from the equation [2] \((h = c = 1)\)

\[
\left( i \frac{\partial}{\partial t} - \hat{H}_{ph} - \frac{q}{m} \hat{p} \hat{A} - \hat{P}_r \right) X = 0. \tag{1}
\]

If interaction of the field with the impurity in the first order with respect to \(q\) is of interest here, it is sufficient to take the approximation

\[
\hat{P}_r = \hat{P}_r + q \frac{\partial \hat{A}}{\partial q}. \tag{2}
\]

The form of the operator \(\hat{P}_r\) with two identical quanta in the state \((\kappa, \lambda)\) in the quasi-resonant approximation \(\kappa = \omega_{mu} \gg |\kappa - \omega_{mu}|\) was found in [2]

\[
\hat{P}_r f = \sqrt{2} \varphi_1 (p) T_r (E | \kappa, \lambda) \varphi_1 (p) f = 2^{-1} \Lambda_{\mu}^{\lambda} \hat{A}_{\mu}^{\lambda} |0 >. \tag{3}
\]

\[
T_r (E | \kappa, \lambda) = \frac{c \Delta_r (E - \omega_{mu} - \varepsilon_i | \kappa, \lambda)}{1 - c \Delta_r (E - \omega_{mu} - \varepsilon_i | \kappa, \lambda) \Delta_r (E - 2 \omega_{mu} - \varepsilon_i | 0)}, \]  

\[
\Delta_r (E | \kappa, \lambda) = \left[ E - \kappa - \frac{c}{E - \omega_{mu}} \right]^{-1} \Delta_r (E | 0) = (E + i 0)^{-1}. \tag{4}
\]
The wave function of the impurity atom in the ground state is denoted by $\Psi_1(\rho)$ and that in the excited state by $\Psi_2(\rho)$. The energy of these states $\varepsilon_{1,2}$ is assumed to be real. Equation (2) corresponds to independent [2] evolution of quanta in the medium. The thermal motion of the atoms is assumed to be insignificant. Taking into account the presence of relaxation processes, resonant frequencies of optical transitions of atoms of the medium are assumed to be complex $\omega_{\text{impu}} = -\gamma/2$. Here $\gamma$ is the total width of the energy level of the excited state. Let $\gamma_T$ denote its radiational component. Equation (2) is found by summing an infinite sequence of Feynmann diagrams, typical representatives of which are shown in Fig. 2. The dashed curves in Fig. 2 correspond to the product of photon Green's functions in Eq. (3) and the Green's functions of the impurity atoms. In the presence of photon interaction with impurity, this product of propagators must be replaced by the propagator of the coupled system. In an energy representation, this step leads, on taking account only of processes of absorption by the impurity, to the substitution

$$
\hat{P}_r = \sum \frac{2}{m} \int \Psi_1^*(\rho) (\rho e^{i\mathcal{K} \rho}) e^{i\mathcal{K} \rho} \hat{P}_r(\rho) d\rho.
$$

The first terms on the right-hand side after summation correspond to the operator $\hat{P}_r$ and the second to the operator $d\hat{P}_r/dq$. The matrix element $p_{ij}^3(\mathcal{K})$ on the Feynmann diagrams will form a cross. Typical diagrams corresponding to this part of the polarization operator follow from the diagrams in Fig. 2 and are shown in Fig. 3. The cross may be inside or outside the ring of the atomic chain. The latter terms lead to $\hat{P}_q/m$ in Eq. (1). If the cross is inside the ring, however, the propagator of phonon vacuum is to the left of it, and the associated chain of atomic loops is unavoidably broken. Thus, the cross may only be found in the ring of the chain that is furthest to the left. This allows all the diagrams to be effectively summed, by the procedure adopted in deriving the expression for $\hat{P}_r$ in [2]. The result of the summation is

$$
q \frac{d\hat{P}_r}{dq} = - \Psi_2(\rho) \Delta_r(E - \omega_{\text{impu}} - \varepsilon_1 | 0) (\mathcal{K} \rho)^{-1/2} p_{ij}^3(\mathcal{K}) T(E | \mathcal{K}, \lambda) | \mathcal{K}, \lambda >.
$$

Now, in the first order in $q$, it follows from Eq. (1) that

$$
X = X^0 + \Psi_2(\rho) | \mathcal{K}, \lambda > \frac{p_{ij}^3(\mathcal{K})}{\mathcal{K} \rho} \int (E - E^{(0)} - \varepsilon_1)^{-1} (E - E^{(1)} - \varepsilon_2)^{-1} \times
$$

$$
\times \left[ \left. \begin{array}{c}
E - \omega_{\text{impu}} - \varepsilon_1 - c
\end{array} \right| (E - \omega_{\text{impu}} - \varepsilon_1 - \mathcal{K})(E - 2\omega_{\text{impu}} - \varepsilon_1 - c)
\right] \exp(-iEt) \frac{dE}{E - E_0} 2\pi i.
$$

Suppose that the interaction of the field with impurity is switched on at time $t = 0$. The wave function of the initial state of the system is taken in the form

$$
X^0(t) = 2^{-1/2} \Psi_1(\rho) \frac{\Delta_r}{\Delta_{\text{impu}}} | 0 > \exp(-iE_0t).
$$

Here, taking account of the relaxation constants, $E_0$ is a complex energy parameter. The possible values of this parameter are determined by the poles of Eq. (3)