10-GeV neutrinos $N_{\nu} = 10^5 \text{ sec}^{-1}$ may be obtained (Doppler reduction in flight time through the crystal $\tau \sin^2 \psi$ is taken into account here). This is evidently sufficient for experimental observation of neutrino emission by relativistic electrons.

LITERATURE CITED


NONLINEAR SIGMA-MODEL WITH A COMPOSITE GAUGE FIELD

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A nonlinear gauge-invariant $\sigma$-model of a general type on a homogeneous space $G/H$ is reformulated in terms of a composite gauge field. In the approximation under consideration, the one-loop effective action of the model is obtained. It is shown that one can choose the parameters in such a way that the theory is both free from ultraviolet divergences and does not lead to a dynamical generation of a composite boson.

1. The study of theories with nonlinear representation of symmetries [1] is in many ways connected to the progress in building supersymmetric field theories, whose Bose sector is a $\sigma$-model. It is known that a wide class of $\sigma$-models (including, in particular, the principal chiral theories and their supersymmetric generalizations) corresponds to renormalizable theories [2]. On the other hand, studying gauge symmetries of such models [3] led to using composite gauge bosons. There are examples of noncompact $\sigma$-models where these bosons, despite their composite nature, acquire dynamics on a quantum level [4, 5]. This is especially important when one thinks of the problem of embedding the low-energy phenomenology into unification schemes, including extended supergravities. Due to the mechanism of dynamical generation of composite boson, one can hope to obtain a sufficient number of gauge fields. Supersymmetric formulations of composite field models are known [6], and the general structure of nonlinear realizations in supersymmetric theories has been studied [7].

In the present paper we consider a $\sigma$-model of a general type with an arbitrarily large gauge symmetry, formulated in terms of a composite gauge field. We want to find out whether the generation of the dynamics of such a field can occur. In Sec. 2, we give a classical description of the model. In Sec. 3, we compute the one-loop effective action and obtain the conditions for the cancellation of divergences. We show that even when these conditions are satisfied, there is no dynamical generation of the composite boson at any value of the mass.

2. The Lagrangian of a general nonlinear $\sigma$-model can be written in a standard form

$$L = \frac{1}{2} g_{ij}(\psi) \partial_\mu \psi^i \partial^\mu \psi^j$$

where $\psi^i, i = 1, \ldots, [M]$ are the coordinates of the homogeneous space $M \equiv G/H$. The metric $g_{ij}(\psi)$ can be taken as
where $e^i_\mu$ is a tetrad. The Lagrangian (1) can be easily rewritten in terms of the elements of the group $g$, using the exponential mapping $g = \exp \phi \ M_m$. Here $T_m$ are the generators of $M$ in the adjoint representation. (We shall generally denote quantities related to the groups $G$, $H$, and $M$, by the Greek indices and indices from the beginning and the middle of the Roman alphabet, accordingly). Now,

$$L = \frac{1}{2} < T_i g^{-1} \partial_j g > A_{ij} < T_j g^{-1} \partial_i g >, \tag{2}$$

where $\langle \ldots \rangle$ denotes the trace on the group $G$. The generators satisfying the commutation relations

$$[T_a, T_b] = i f_{abc} T_c, \quad (T_a)_{ij} = i f_{ijk},$$

will be normalized by the condition

$$\langle T_a T_b \rangle = f_{abc} f_{bde} i = 5_{ac}.$$

The structure constants are entirely antisymmetric, and $f_{ijk} = 0$. The explicit expressions for the tetrads follow from Eq. (2) and can be found in [8]. Our notation coincides with this of [8].

The requirement that Eqs. (1) or (2) be invariant under the transformations from the group $G$,

$$\varphi' = \varphi + R_a (\varphi) e^a$$

has the form

$$D_a g_{ij} \equiv R^x_a g_{ij, \kappa} + R^x_{ij} g_{x, \kappa} + R^x_{ij} g_{x, \kappa} = 0,$$

where $D_a$ is the Lie derivative:

$$D_a R^x_a \equiv R^x_a R^x_{\kappa, \kappa} - R^x_{\kappa, x} R^x_\beta = f_{a_b c} R^c_a.$$

This requirement leads to restrictions on the metric [8]:

$$A_{ij}(\varphi) = A_{ij}(0) = \text{const}, \quad [T_a, A_{ij}] = 0. \tag{4}$$

If $T_a$ are irreducible on $M$, then by virtue of the Schur lemma, $A_{ij} = \lambda \delta_{ij}$.

In the case where a part of the parameters in Eq. (3) corresponds to gauge transformations,

$$\{e^m\} = \{e^x(x), e^x(\varphi)\}, \quad \{T_m\} \subset R, \quad \{T_x\} \subset N,$$

the fact that $G$ is semisimple and the parameters $R$ are local results in the following commutation relations:

$$[H, H] \subset H, \quad [H, N] \subset N, \quad [R, R] \subset R,$$

$$[N, N] \subset H \oplus N, \quad [H, R] = [N, R] = 0 \tag{5}$$

(for the sake of simplicity, we denote algebras by the same letters as the corresponding groups). As could be expected, Eq. (5) means that the gauge transformations form an ideal in $G$,

$$G = K \otimes R, \quad N = K \otimes H.$$ 

The requirement of gauge invariance of the action (1) or (2) leads to a restriction on the metric

$$R^x_{ij} g_{ij}(\varphi) = 0,$$

whence, using the explicit expression for the generators via the group elements [8] and the commutation relations (5), one can obtain

$$A_{mn} = \begin{pmatrix} 0 & 0 \\ 0 & \frac{1}{A_{mn}} \end{pmatrix}, \tag{6}$$

i.e.,

$$L = \frac{1}{2} < T_m g^{-1} \partial_\mu g > A_{mn} < T_n g^{-1} \partial_\mu g >. \tag{7}$$