CALCULATION OF THE RADIATION FIELD FROM AN EXTENDED SOURCE

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The indications of a detector with sensitivity function \( D(r, p) \) \((p = \Omega, E)\) are considered in a radiation field created by a source with distribution density \( S(r, p) \). If this density changes slowly at distances of the particle path length, the solution may be written in the form of a series, the terms of which are expressible in terms of Legendre transforms of the source distribution density and the equilibrium Green function.

The indication of a detector with sensitivity function \( D(r, p) \) \((p = \Omega, E\) are the direction of motion and particle energy) in the radiation field from a source with distribution density \( S(r, p) \) can be written in the form

\[
I = \int dr \int dp D(r, p) \int dr' \int dp' G(r, p; r', p') S(r', p'),
\]

where \( G(r, p; r', p') \) is a Green function of the transfer equation satisfying the kinetic equation

\[
\Omega G(r, p; r', p') + \Sigma(E) G(r, p; r', p') - \int dp'' \Sigma_\nu(p'' \to p') \times
\]

\[
G(r, p'; r', p') = \delta(p - p'),
\]

where \( \Sigma \) is the interaction section and \( \Sigma_\nu \) is the differential scattering section.

If the medium in which the radiation propagates is homogeneous, and the source distribution density changes little at distances of the order of the particle path length, then by expanding \( S(r'; p') \) in a series \( S(r', p') \approx S(r, p') - (r - r') \psi S(r, p') \), Eq. (1) can be transformed to

\[
I = \int dr \int dp D(r, p) \int dp' R(p; p') S(r, p') - \int dr \int dp D(r, p) \times
\]

\[
\int dp' R(p; p') \psi S(r, p'),
\]

where

\[
G(p; p') = \int G(r, p; r', p') dr', \quad R(p; p') = \int (r - r') G(r, p; r', p') dr'.
\]

The first term in Eq. (3) corresponds to the approximation in which spatial displacement of particles is neglected, or in other words, it describes the detector indications under equilibrium conditions \([1, 2]\) where the number of particles departing from the vicinity of the point \( r \) is equal to the number of particles entering the vicinity. The second term describes deviation from the equilibrium state, defining the value of the correction connected with the finite value of the particle path length. For a homogeneous source this correction is obviously equal to zero.

The characteristics of the transfer process necessary for calculation of detector readings with Eq. (3) are the functions \( G(p; p') \) and \( R(p; p') \), whose calculation is simpler than that of the Green function \( G(r, p; r', p') \) for general equation (1). They describe kinetic equations which may be obtained from Eq. (2).

Integrating all terms of Eq. (2) over the coordinates \( r' \) and considering the fact that the radiation flux from a homogeneous source is independent of coordinate, we obtain

\[
\Sigma(E) G(p; p') - \int dp'' \Sigma_\nu(p'' \to p) G(p'; p') = \delta(p - p').
\]

Similarly, multiplying all terms of Eq. (2) by \( r - r' \) and integrating over \( r' \), we obtain

\[
\Sigma(E) R(p; p') - \int dp'' \Sigma_\nu(p'' \to p) R(p'; p') = \Omega G(p; p').
\]


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The right side of this equation is obtained by transforming the gradient term of the equation by integration by parts with consideration of the fact that in a homogeneous medium the Green function depends on the difference \( r - r' \), in consequence of which \( \nabla G(r, p; r', p') = - \nabla' G(r, p; r', p') \).

From Eqs. (4) and (5) it is evident that \( G(p; p') \) is a Green function of Eq. (5) so that

\[
R(p; p') = \int dp' G(p; p') \Omega'' G(p''; p').
\]  

(6)

If the detector is isotropic, i.e., \( D(r, p) = D(r, E) \), then in Eq. (3) we may integrate over \( \Omega \). From the physical meaning of \( G(p; p') \) it follows that the integral

\[
G_0(E; E') = \int \Omega'' G(p; p') d\Omega
\]  

(7)

is independent of \( \Omega' \). Integrating both parts of Eq. (6) over \( \Omega \) with consideration of Eq. (7) we obtain

\[
\int R(p; p') d\Omega = \int dE'' G_0(E; E') \int d\Omega'' \Omega'' G(p''; p').
\]  

(8)

It can easily be seen that the function \( G(p; p') \) depends on \( \cos \theta = \Omega \Omega' \), so that the inner integral in Eq. (8) is a vector directed along \( \Omega' \), i.e.,

\[
\int \Omega'' G(p''; p') d\Omega'' = \Omega' G_1(E'; E'),
\]  

(9)

where

\[
G_1(E; E') = \int (\Omega') G(p; p') d\Omega.
\]

We note that \( G_0(E, E') \) and \( G_1(E, E') \) are the first coefficients of an expansion of the function \( G(p, p') \) in Legendre polynomials:

\[
G(p; p') = \sum_{l=0}^{\infty} \frac{2l + 1}{4\pi} G_l(E; E') P_l(\cos \theta), \quad G_1(E; E') = \int G(p; p') P_1(\cos \theta) d\Omega
\]

and satisfy the kinetic equations

\[
\sum \left( E' \right) G_1(E; E') - \int dE'' \sum_{l=1} \left( E'' \rightarrow E \right) G_1(E''; E') = \delta(E - E'),
\]  

(10)

where

\[
\sum_{l=1} \left( E' \rightarrow E \right) = \int \sum_{l=1} \left( p' \rightarrow p \right) P_l(\cos \theta) d\Omega
\]

(11)

is the transformant of the differential scattering section.

Substituting Eqs. (7), (8), (9), in Eq. (3), we obtain

\[
I = \int dr dE D(r, E) \int dE' G_0(E; E') S_0(r, E') - \int dr dE D(r, E) \times
\]

(12)

\[
\int dE' \int dE'' G_0(E; E'') G_1(E''; E') \text{div} S(r, E'),
\]

where

\[
S_0(r, E) = \int S(r, p) d\Omega, \quad S(r, E) = \int \Omega S(r, p) d\Omega.
\]  

(13)

From Eq. (13) it is evident that for an isotropic source \( S(r, E) = 0 \), so that the second term in Eq. (12) is also equal to zero.

Equation (12) simplifies if the sensitivity function \( D \) is independent of coordinate. In that case

\[
I = \int \int I_0(E) S_0(\Delta V, E) dE - \int \int I_0(E) \int dE' G_1(E; E') \int \int dS(r, E') d\Omega
\]

(14)

where \( \Delta V \) is the sensitive volume of the detector; \( \sigma \) is the detector surface; and \( \mathbf{n} \) is the normal to the elementary area \( ds \),

\[
S_0(\Delta V, E) = \int S_0(r, E) dr, \quad I_0(E) = \int D(E') G_0(E', E') dE'.
\]