THEORY OF A GRAVITATIONAL GAUGE FIELD
WITH LOCALIZATION OF THE DE SITTER GROUP

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A de Sitter-invariant gauge theory is formulated for the case where a 40-component de Sitter A-field is present. It is shown that the theory coincides with the Poincaré-invariant gauge theory in a space with torsion with a cosmological term. Two other versions of a de Sitter-invariant theory are also discussed: the first is a metric theory of gravitation in a Riemann space; the second is a de Sitter-invariant generalization of the tetrad theory of gravitation in a space of absolute parallelism.

1. Recently, a number of papers (see, for example, [1–6]) have been devoted to studies of gauge theories with localization of the de Sitter group. For applications, a centrally symmetric static field (CSSF) and a homogeneous isotropic cosmological model (HICM) are of special interest. Thus, we shall start from the de Sitter universe (dSU) in the following two basic forms [6]:

\begin{align}
1) \quad ds^2_0 &= e^{\gamma_0} c^2 dt^2 - e^{\gamma_0} \delta_{mn} dx^m dx^n, \\
2) \quad e^{\gamma_0} &= e^{\phi_0} \left(1 - \frac{r^2}{4K^2}\right), \quad \phi_0 = \left(1 + \frac{r^2}{4K^2}\right)^{-\frac{1}{2}};
\end{align}

where \( K = \text{const} > 0 \) is the radius of curvature of the dSU. After the introduction of the de Sitter A-field \( A_B(\gamma)^{(\alpha)}_\mu \), physical space-time acquires a Riemann metric [6]:
where
\[ e_\alpha(P_{ij}(Q)) = \Lambda^{(1)}_{(P_i)(Q_j)}(t_0), \quad b_{ij}^{(S)} = \partial_\mu e_{ij}^{(S)}. \]

Starting from (4) we can determine the Riemann–Cartan metric connection in a de Sitter-invariant theory:
\[ \Gamma_{\mu}^{\nu} = e_\nu(P_{ij}(Q)) b^{\mu^{(S)}} \frac{\partial}{\partial x^\mu} + \frac{1}{2} g^{\nu \gamma} \left\{ 2 b^{(P)}_\mu b^{(Q)}_\nu \partial_\gamma e_{\alpha}(P_{ij}(Q)) - 4 b^{(S)}_\mu b^{(Q)}_\nu \partial_\gamma e_{\alpha}(P_{ij}(Q)) \right\}, \]
\[ \Gamma_{\mu}^{\nu} = \partial_\nu b^{(Q)}_\mu - \frac{1}{2} A^{(Q);S} b_{\mu \nu}^{(S)}, \]
\[ \Gamma_{\mu} e_{\nu}(P_{ij}(Q)) = \partial_\mu e_{\nu}(P_{ij}(Q)) + \frac{2}{K} e_{(i)(j)} A^{(Q);S} b_{\mu \nu}^{(S)}. \]

The corresponding torsion tensor is given by
\[ C_{\mu^\nu} = 2 F_{\nu i}(Q) b^{(S)} + \frac{1}{K} F_{\nu i}(Q) b^{(S)} N_{(i)}^{(j)} e_{(j)}^{(S)} \]
where
\[ F_{\nu i}(Q) = 2 \partial_\nu A^{(C)(Q)}, + \frac{2}{K} A^{(C);S} A_{(i)}^{(S)} (Q) \]
is the A-field strength tensor. We also introduce a tensor
\[ \rho_{\alpha \beta} = R_{\nu i j}^{\alpha \beta} + \frac{1}{K} (g_{\nu i} g_{\beta j} - g_{\nu j} g_{\beta i}), \]
where \( R_{\alpha \beta \gamma \delta} \) is the curvature tensor. In the de Sitter universe we have \( C_{\mu^0} = 0 \) and \( \rho_{\alpha \beta} = 0 \) after turning off the A-field. Using the standard expression for the matter Lagrangian in the Riemann–Cartan space \( U_\alpha \) (see, e.g., [7-10]) we find the matter sources for the A-field:
\[ J_{(i)(j)}^{(Q)} = \frac{2K}{V - g} \frac{\delta (V - g L_{\alpha \beta})}{\delta A_{(i)(j)}^{(Q)}} = 2 \partial_{(i)} A^{(C)(Q)} b_{(i)}^{(Q)} \left\{ ! T_{(i)(j)}^{(Q)} + \Sigma^{\mu \nu} \times \right\}
\times \left[ b^{(Q)}_\mu b^{(P)}_\nu \partial_\gamma e_{(i)(j)}^{(Q)} \right] + \frac{1}{2} b^{(P)}_\mu b^{(Q)}_\nu \partial_\gamma e_{(i)(j)}^{(Q)} \]
where
\[ \Sigma^{\mu \nu} = \frac{1}{2} (S^{\mu \nu}; \cdot S^{\gamma \sigma}; + S^{\gamma \sigma}; \cdot S^{\nu \sigma}), \]
and \( T_{\mu \nu}^{(C)} \) and \( S_{\mu \lambda \alpha} \) are the canonical definitions of the energy-momentum tensor and the spin angular momentum tensor in \( U_\alpha \). By analogy with the Poincaré-invariant gauge theory (PGT) in a space with torsion (see, e.g., [7-10]), we will assume that the Lagrangian for the A-field is quadratic with respect to geometrical tensors of curvature and torsion:
\[ L_{\alpha} = i_{\nu} C^{(i \lambda)} - i_{\nu} C^{(i \lambda)} + i_{\nu} C_{\alpha}^{(i \lambda)} + i_{\nu} P_{\lambda}^{(i \lambda)} + i_{\nu} P_{\lambda}^{(i \lambda)} + i_{\nu} P_{\lambda}^{(i \lambda)} + i_{\nu} P_{\lambda}^{(i \lambda)} + i_{\nu} P_{\lambda}^{(i \lambda)} + i_{\nu} P_{\lambda}^{(i \lambda)}, \]
where
\[ C_{\alpha} = C^{(i \lambda)} \quad P_{\lambda} = g^{\alpha \gamma} P_{\lambda \gamma}, \quad P_{\lambda}^{(i \lambda)} = g^{\alpha \gamma} P_{\lambda \gamma}. \]
A direct calculation shows that:
\[ G_{(i)(j)}^{(Q)} = 2 \partial_{(i)} (V - g L_{\alpha \beta}) g_{(i)(j)}^{(Q)} A_{(i)(j)}^{(Q)} b_{(i)}^{(Q)}(Q) \times \left\{ K^\mu_{(i)} + 2 n^{\mu \nu} \left[ b^{(Q)}_\mu b^{(P)}_\nu \partial_\lambda e_{(i)(j)}^{(Q)} - \frac{1}{2} b^{(P)}_\mu b^{(Q)}_\nu \partial_\lambda e_{(i)(j)}^{(Q)} \right] \right\} + \frac{1}{2} b^{(P)}_\mu b^{(Q)}_\nu \partial_\lambda e_{(i)(j)}^{(Q)} \times \]
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