INTENSITY OF \( \gamma \) RADIATION BACK-SCATTERED AS A RESULT OF THE MÖSSBAUER EFFECT

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In the geometry of total backscattering, the intensity of scattered Mössbauer \( \gamma \) radiation is calculated. The contribution of various scattering channels to the spectrum is discussed: resonant and nonresonant, elastic and inelastic. Scattering at samples of different thickness is considered. Only one-time scattering of \( \gamma \) quanta in the material is taken into account.

Mössbauer spectroscopy is a widespread method of investigating the properties of solids and the character of chemical bonds. Recently, it has come into increasing use for monitoring technological processes and in various applied investigations. The requirement of in situ investigation, as a rule, rules out the application of Mössbauer spectroscopy in transmission and encourages the development of various methods of spectroscopy in scattering.

Some methodological questions associated with the search for optimal conditions for obtaining maximum amplitude of the spectral lines in scattering were considered in \([1, 2]\) for a scattering geometry in which the collimated flux of Mössbauer \( \gamma \) quanta is incident on a sample and reflected at a definite angle.

Experiment shows that, for samples with a very small content of the resonant isotope, the spectrum from a sample of homogeneous composition may in some cases take the form not of peaks but of troughs in intensity, similar in external form to the spectra observed in transmission spectroscopy. This is because the role of nonresonant scattering channels increases with reduction in content of the resonant isotope, and there is gradual transformation of the amplitude of the spectral lines, culminating in the appearance of intensity troughs. This situation was analyzed in \([3]\) for a series of hypothetical cases. Some questions associated with the form of the spectra in total-backscattering geometry were considered for the first time in \([4, 5]\), in the simplest case of a sample of homogeneous composition. The significantly more complex case of two-layer samples was considered in \([6]\). Some problems of Mössbauer spectroscopy associated with the recording of K-characteristic x-rays arising in resonant scattering were investigated in \([7]\).

In these works, no analytical expressions permitting sufficiently simple estimates of the intensity of scattered radiation and the relative and absolute contribution of various scattering channels were obtained. The present work is devoted to the solution of these problems for experiments in total-backscattering geometry, when a proportional counter records only the \( \gamma \) radiation.

Consider a sample at which Mössbauer radiation with a Lorentzian distribution of the energy \( E \) is incident; the width of the distribution is \( \Gamma \) and it is centered at energy \( \varepsilon \)

\[
L(E-\varepsilon) = (\Gamma/2\pi)/((\Gamma/2)^2 + (E-\varepsilon)^2).
\]

Suppose that the width of the nuclear levels in the source and the sample is the same; the unit of measurement is the halfwidth of the nuclear level \( \Gamma/2 = 1 \); and the intensity of the source is normalized to unity. With source motion at a rate \( v \) relative to the scatterer, \( \varepsilon = E_0 + \delta - E_0v/c \), where \( E_0 \) is the position of the center of the line of the motionless source; \( c \) is the velocity of light; \( \delta \) is the chemical shift. With probability \( f' \), the resonantly scattered \( \gamma \) quantum does not lose its energy in recoil and may then be resonantly re-scattered. There are always also nonresonant scattering channels: Rayleigh scattering, Compton scattering, and the photo effect. For any scattering channel \( i \), the contribution to the \( \gamma \)-ray intensity recorded by the detector may be written in the form

Here \( P_i(E) \) is the probability that a \( \gamma \) quantum of energy \( E \) is scattered in channel \( i \) and the scattered \( \gamma \) quantum then reaches the sample surface; \( C_i \) is the weighting factor of the channel. For a resonant channel at whose output there appears a \( \gamma \) quantum which does not lose its energy in recoil, \( C = f'/(1 + \alpha) \), where \( \alpha \) is the internal-conversion coefficient.

It is expedient to write the spectral form as in Eq. (1), since a convolution of two functions is obtained: one describes processes in the source only, and the other processes in the sample only. The chemical composition of the sample, its structure and thickness, the interaction processes between a quantum of energy \( E \) and the scatterer material — all this determines the specific form of \( P(E) \). In this sense, \( P(E) \) may be regarded as the distinctive fingerprint of the sample. Consequently, the fairly complex problem of analyzing the form of the spectrum — i.e., the function \( I(\varepsilon) \) — may be replaced first of all by the analysis of the function \( P(E) \). Note here that \( P(E) \) is determined by the scatterer and does not depend on the velocity of relative motion.

The probability \( P_i(E) \) includes, generally speaking, all the possible processes occurring with the emission of a \( \gamma \) quantum inside the material following scattering in the channel \( i \). However, in the first approximation, the rescattering of this radiation is not taken into account. In the second approximation, the probability of repeated scattering of the radiation in the material must be taken into account. Higher-order approximations may play a pronounced role only in some special cases for resonant processes. The contribution of repeated one-time scattering increases with depth of the layer in which scattering occurs, and increases with increase in concentration of M"ossbauer nuclei. Calculations show that it is no more than 10\% of the accurate value of \( P_i(E) \). Processes of repeated double scattering must be taken into account only in those cases where an accuracy of solution better than 1\% is required.

In the first approximation (i.e., without taking account of rescattering events) the probability \( P_i(E) \) takes the form

\[
P_i(E) = \int_0^{\pi/2} \int_0^d \exp \left( -\mu_0(E) x \right) \mu_i(E) \exp \left( -\nu_i(E) \frac{x}{\cos \theta} \right) \times \nonumber\]
\[
\times w(\theta) 2\pi \sin \theta d\theta dx, \nonumber\]

where \( d \) is the sample thickness; \( \mu_0(E) \) is the total linear coefficient of interaction of the M"ossbauer radiation with the sample material; \( \mu_i(E) \) is the linear interaction coefficient corresponding to scattering channel \( i \); \( \nu_i(E) \) determines the attenuation of the radiation arising in channel \( i \) on interaction with the scatterer material. In isotropic scattering, the probability density of scattering at an angle \( \theta \) is \( w(\theta) = 1/4\pi \). For simplicity, only this case is considered below.

It is simple to show that, after switching to the variable \( t = 1/\cos \theta \) and introducing the integral exponentials \( E_n(y) = \int_0^\infty e^{-yt} dt / t^n \), Eq. (2) takes the form

\[
P_i(E) = \frac{\nu_i(E)}{2\nu_i(E)} \int_0^{\pi/2} \exp \left( -\frac{\nu_0(E)}{\nu_i(E)} y \right) E_2(y) dy = \nonumber\]
\[
= \frac{\nu_i(E)}{2\nu_i(E)} \hat{E}_2(\nu_0(E) / \nu_i(E), \nu_i). \quad (3) \nonumber\]

Analysis of \( P_i(E) \) now reduces practically to the analysis of the function \( \hat{E}_2(\alpha, b) \) appearing in Eq. (3).

For arbitrary \( \alpha \) and \( b \)

\[
\hat{E}_2(\alpha, b) = \hat{E}_2(\alpha, \infty) - \frac{1}{\alpha^2} \{E_1(b(1 + a)) - e^{-\alpha b} [(1 + ab) E_1(b) - ae^{-b}] \}, \quad (4) \nonumber\]