The contours $\Gamma_a$ and $\Gamma_b$ are displayed in Fig. 1, respectively, for $u^0 > u_0^0$ and

$$u^0 < u_0^0 \left( a_0 = a_0 = \frac{1}{2} \ln \frac{A_\lambda(u^0) - A_\lambda(-\infty)}{A_\lambda(u^0) - A_\lambda(-\infty)} - i\pi \right).$$

In the particular case as $A_\lambda(u^0) \to \text{const}$ we obtain representations for $\tilde{S}, \tilde{S}^0, \text{and } \tilde{S}$ in a constant electrical field and in a plane wave field from (22), (23), and (24), and which agree with those found earlier in [3, 6].

LITERATURE CITED


GREEN'S ELECTRON FUNCTION IN A QUANTIZED PLANE WAVE FIELD

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It is shown that the Green's function of an electron that interacts with a quantized plane wave can be expressed in terms of the corresponding Green's function of a scalar particle. By using the known expression for the Green's function of a scalar particle, an integral representation is found with respect to the intrinsic time for the Green's electron function in a quantized plane wave of arbitrary form.

1. The problem of electron motion in the field of a free quantized monochromatic electromagnetic wave with which the interaction is taken into account exactly, was first solved in [1]. The solutions found in that paper were then extended to more complex cases by different authors [2-6]. The limits of applicability of such a model were then established in [7] within the framework of exact quantum electrodynamics.

The Green's function of a scalar particle interacting with a quantized plane electromagnetic wave field was found in [8], where no constraints were imposed on the wave. A relation between the electron and the scalar particle Green's function in a quantized electromagnetic field of arbitrary form is established in this paper and the Green's electron function is calculated. Exactly as in [8], the Green's function is represented in the form of a contour integral over the intrinsic time.

2. We select the $x^3$ axis along the direction of plane wave propagation. The 4-potential of the wave will then depend on the space-time variables in the combination $x^0 - x^3$. It is convenient to introduce the new coordinates

$$u^0 = x^0 - x^3, \quad u^1 = x^1, \quad u^2 = x^2, \quad u^3 = x^0 + x^3. \quad (1)$$

We shall denote the components of any vector $A$ relative to the new coordinate system by $\tilde{A}^\mu$ in contrast to the Cartesian $A^\mu$.

The Green's function of an electron interacting with a quantized plane wave field will satisfy the equation

$$
(\gamma^\mu \tilde{P}_\mu - m) G(x, x'; \xi, \xi') = -\delta^{(4)}(x - x') \delta(\xi - \xi'),
$$

(2)

where

$$
\delta(\xi - \xi') = \prod_{\lambda} (\delta(\xi_\lambda - \xi'_\lambda), \tilde{P}_\mu = i\partial_\mu - e\tilde{A}(x, \xi), \partial_\mu = \partial / \partial x^\mu, \hbar = c = 1;
$$

$$
\tilde{A}^\mu(x, \xi) = \sum_{\lambda, \lambda'} \left( \frac{2\pi}{Vz_1} \right)^{1/2} e_{\lambda'}^\nu (c_{\lambda'} e^{-i\xi_\lambda^\nu} + c_{\lambda'}^* e^{i\xi_\lambda^\nu})
$$

(3)

is the operator-potential of the quantized wave field, $V$ is the normalizing volume of the field

$$
c_{\lambda} = 2^{-1/2} \left( \xi_{\lambda} - \frac{\partial}{\partial \xi_{\lambda}} \right), \quad c_{\lambda'} = 2^{-1/2} \left( \xi_{\lambda'} + \frac{\partial}{\partial \xi_{\lambda'}} \right)
$$

are photon generation and annihilation operators in a coordinate representation with the frequency $\omega$ and polarization $\lambda = 1, 2$, $\xi_{4\lambda}$ is the field variable, and $e_\mu^\lambda = (0, e_{\lambda 1}, e_{\lambda 2}, 0)$ is the 4-vector of linear photon polarization.

The solution of (2) can be found by the inverse operator method [9]

$$
G(x, x'; \xi, \xi') = -\frac{1}{\gamma^\lambda \gamma^0 (\gamma^\mu \tilde{P}_\mu - m)^2 - m^2 + i\epsilon}
$$

$$
= \left( \frac{\gamma^\lambda \gamma^\mu \tilde{P}_\mu + m}{(\gamma^\mu \tilde{P}_\mu)^2 - m^2 + i\epsilon} \right) d\tilde{S} e^{-(\gamma^\mu \tilde{P}_\mu)^2 - m^2 + i\epsilon} \delta^{(4)}(x - x') \delta(\xi - \xi').
$$

(4)

However, by starting from (2) it is simpler to express $G$ in terms of the scalar particle Green's function, and then to determine $G$ by using the known expression for it.

To this end, we introduce the projection operators $p(-)$ and $p(+)$

$$
p(-) = \frac{1}{4} \tilde{\gamma}^0, \quad p(+) = \frac{1}{4} \tilde{\gamma}^0, \quad p(-) + p(+) = 1,
$$

$$
p(\pm)p(\pm) = p(\pm), \quad p(\pm)p(\mp) = 0.
$$

(5)

By using $p(-)$ and $p(+)$ we write $G$ in the form

$$
G = G(+) + G(-), \quad G(\pm) = p(\pm)G.
$$

(6)

Multiplying (2) on the left by the matrices $\tilde{\gamma}^0$ and $\tilde{\gamma}^\lambda$, we obtain a system of equations in the functions $G(\lambda)$ and $G(-)$:

$$
4i\partial_\lambda G(-)(x, x'; \xi, \xi') = -\tilde{\gamma}^0 \left[ (\gamma^\mu \tilde{P}_\mu - m) G(+) (x, x'; \xi, \xi') + \delta^{(4)}(x - x') \delta(\xi - \xi') \right],
$$

(7)

$$
4i\partial_\lambda G(+) (x, x'; \xi, \xi') = -\tilde{\gamma}^\lambda \left[ (\gamma^\mu \tilde{P}_\mu - m) G(-)(x, x'; \xi, \xi') + \delta^{(4)}(x - x') \delta(\xi - \xi') \right],
$$

(8)

$$
j = 1, 2.
$$

The last equation can be solved formally for $G(\lambda)$:

$$
G(\lambda)(x, x'; \xi, \xi') = -\tilde{\gamma}^\lambda (4i\partial_\lambda)^{-1} \left[ (\gamma^\mu \tilde{P}_\mu - m) G(-)(x, x'; \xi, \xi') + \delta^{(4)}(x - x') \delta(\xi - \xi') \right],
$$

(9)

where $(i\partial_\lambda)^{-1}$ is the inverse operator to $i\partial_\lambda$. To determine the operator $(i\partial_\lambda)^{-1}$, we note that the expansion of the scalar particle Green's function $D$ in the field (3) in a Fourier integral in the variable $u^\mu$ has the form