A METHOD FOR CALCULATING SPECIFIC ACTIVITY
OF FISSION PRODUCTS

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An algorithm is described for determination of specific activity of any member of a linear decay chain. The algorithm permits calculation of the activity of fission products for all three possible accumulation modes: 1) coexistence, i.e., irradiation time in a reactor; 2) delay after preceding coexistence; 3) delay after instantaneous fission. In comparison with previous studies in this area the proposed method is more suitable for use with computers, both from the point of view of calculating input parameters and accuracy, as well as in convenience of programming.

The formulas usually used for calculation of accumulation of activity of fission products along decay chains [1, 2] are inconvenient for practical application, especially in the examination of relatively long chains. The formulas presented usually reflect the accumulation of the third or fourth member of the chain and under certain limitations related to the presence of independent outputs, the preceding members of the chain. Moreover, they do not consider fuel combustion.

The present study will propose an algorithm with no limitations related to chain length and the presence of independent outputs of precursors.

We will examine the most general case, when accumulation of fission products occurs in the operation of a reactor at constant power. It is known that in this case the number of $^{235}$U and $^{238}$U fissions decreases with time as $e^{-at}$, while the number of fissions of $^{239}$Pu increases as $1 - e^{-bt}$ [3] (we consider a thermal neutron reactor).

For calculation, the branched chains are broken into several linear chains. The decay equation for all members of a linear chain of length $n$ elements is written as follows:

\[
\frac{dQ_i(t)}{dt} = \lambda_i x_i p(t) - \lambda_i Q_i(t),
\]

\[
\frac{dQ_{i-1}(t)}{dt} = \lambda_i x_i p(t) - \lambda_{i-1} Q_{i-1}(t) + \lambda_i Q_i(t).
\]

Initial conditions are

\[
Q_i(0) = Q_{0i},
\]

\[
Q_i(0) = Q_{0i}, \quad i = 1, 2, 3, ..., i, ..., n.
\]

Here $i$ is the number of the member of the linear chain, $Q_i(t)$ is the activity of the $i$-th member at time $t$; $x_i$ is the independent output of the $i$-th isotope during fission; $p(t)$ is the number of fissions per unit time.

As was indicated above, $p(t)$ may have the form:

1. $p_1(t) = p_0 e^{-at}$,
2. $p_2(t) = p_0 (1 - e^{-bt})$.

We will examine case I. From the first equation of system (1) a solution for the activity of the first member of the chain follows

\[
Q_1(t) = \frac{\lambda_1}{\lambda_i - \alpha} x_1 p_0 e^{-\alpha t} + \left( Q_{01} - \frac{\lambda_1}{\lambda_i - \alpha} x_1 p_0 \right) e^{-\beta t}.
\]

(3)

We will seek a solution for \( i \neq 1 \) in the form

\[
Q_i(t) = b_i e^{-\alpha t} + \sum_{j=1}^{i} C_{ij} e^{-\lambda_j t}.
\]

(4)

Substituting Eq. (4) in Eq. (1) and performing the corresponding transformations, we obtain

\[
\sum_{j=1}^{i} \left[ C_{ij} (\lambda_i - \lambda_j) - C_{i-1,j} \lambda_i \right] e^{-\lambda_j t} = \left[ -b_i (\lambda_i - \alpha) + b_{i-1} \lambda_i + \lambda_i \lambda_j x_0 \right] e^{-\alpha t}.
\]

(5)

In order that Eq. (5) be identically satisfied for all \( t \) it is necessary and sufficient that all coefficients of each exponent be equal to zero.

From this follows the recurrent relationship for \( C_{ij} \) and \( b_i \)

\[
C_{ij} = C_{i-1,j} \frac{\lambda_i}{\lambda_i - \lambda_j}, \quad i \neq j,
\]

(6)

\[
b_i = \frac{\lambda_i}{\lambda_i - \alpha} \left( b_{i-1} + \lambda_i x_0 \right).
\]

(7)

For \( i = 1 \) from Eq. (7) we obtain

\[
b_1 = \frac{\lambda_1}{\lambda_1 - \alpha} x_1 p_0.
\]

(8)

hence

\[
b_1 = p_0 \sum_{k=1}^{i} x_k \prod_{j=1}^{i} \frac{\lambda_j}{\lambda_j - \alpha}.
\]

(9)

Since \( j \leq i \), then \( n(n - 1)/2 \) coefficients \( C_{ij} \) are required. The relationships of Eq. (6) give \( n(n - 1)/2 \) conditions.

The remaining \( n \) conditions can be obtained from the initial conditions of Eq. (2):

\[
b_1 + \sum_{j=1}^{i} C_{ij} = Q_0.
\]

(10)

Thus, the system for calculation of the coefficients for Eq. (4) is as follows

\[
b_i = p_0 \sum_{k=1}^{i} x_k \prod_{j=1}^{i} \frac{\lambda_j}{\lambda_j - \alpha},
\]

(11)

\[
C_{11} = Q_{01} - b_1.
\]

(12)

\[
C_{ii} = C_{i-1,i} \frac{\lambda_i}{\lambda_i - \lambda_1}, \quad i = 2, 3, \ldots, n,
\]

(13)

\[
C_{22} = Q_{02} - b_2 - C_{21},
\]

(14)

\[
C_{32} = C_{i-1,2} \frac{\lambda_i}{\lambda_i - \lambda_2}, \quad i = 3, 4, \ldots, n,
\]

(15)

and so on up to \( C_{nn} \).

For case II in an analogous manner we obtain the solution

\[
Q_i(t) = a_i - b_ie^{-\beta t} + \sum_{j=1}^{i} C_{ij} e^{-\lambda_j t},
\]

(16)