A method is developed for recovering the density function of annihilating electron-positron pairs from the angular correlations of the annihilation gamma rays. On the basis of this positron-annihilation mechanism, the electron density functions of the matter are determined from the time spectra of the positron lifetime and the angular distributions of the annihilation gamma rays.

The method of positron annihilation in matter has been used recently to investigate the electron structure of the outer shells of atoms and ions and the momentum distribution of free electrons [1, 2]. The use of the method is based on recovering the momentum distribution function from observations of the angular correlations or the Doppler broadening of the annihilation spectrum. In [3], the momentum spectrum of the electron-positron pairs was recovered from the angular correlations for point geometry of the source and the detectors employed without allowance for the actual geometry of the experiment. A simple computational formula was obtained relating the required momentum distribution to \( I(\alpha) \), the counting rate of coincidences of annihilation gamma rays:

\[
\rho \left( \frac{p_z}{2m_e c} \right) = \frac{1}{\alpha} \frac{dI(\alpha)}{d\alpha},
\]

where \( \alpha \) is the angular coordinate of the point detector and \( p_z \) is the momentum component of the electron on which the annihilation occurred. However, in [3] no allowance was made for the distribution of the probability function for detecting an annihilation event, \( P(\nu, \alpha) \), and in the general case this depends on the total momentum of the electron-positron pair. In the region of high momenta, \( \rho(p_z/2m_e c) \approx \rho(p/2m_e c) \), which enables one to obtain the momentum distribution of the annihilating pairs, but in the limit \( p \to 0 \) this approximation is not well satisfied.

The detection of the angular correlations of the annihilation radiation is considered more rigorously and consistently in [4], in which allowance is made for the probability distribution of event detection. For the real point-linear geometry of the experiment, the solution of the integral equation is written down as an infinite series whose first term is equal to Stewart's function [3]. However, the series converges non-uniformly and at low momenta the convergence rapidly deteriorates. In this paper, we propose a more accurate method of recovering the density function of annihilating electron-positron pairs from the angular correlations of the gamma rays. We propose a model of the annihilation mechanism which enables one to determine the electron density function of matter on the basis of the results of time measurements.

---

*The material of this paper was presented at the Fourth International Conference on the Density of Electrons, Spins, and Moments (Minsk, August, 1973).
Recov€ry of the Momentum Distribution Function of Electron–Positron Pairs

We consider a point–spatial geometry of the measurement of the angular correlation of gamma rays which reflects fairly fully the conditions of a real experiment; the sample is a point, while the windows of the detector are surfaces of finite area. We project the images of the detectors onto the surface of a sphere with radius numerically equal to the distance that light traverses in unit time. We place the center of the sphere at the point $o$, at which the investigated sample is (Fig. 1). If the areas of the windows of the detector are $s = a \times b$, the areas of their shadows are accordingly $s' = a' \times b'$, where $a' = l a / d$, $b' = l b / d$. The curvature of the surface of the sphere can be ignored.

The problem of the emission of two gamma rays at an angle $\beta = \pi - 2\alpha$ from the point $o$ reduces to the solution of the system of two equations

$$
\begin{align*}
\nu_1 + \nu_2 &= 2m_e v, \\
n_1 c + n_2 c &= 2m_e c^2,
\end{align*}
$$

where $\nu_1$ and $\nu_2$ are the wave vectors of the annihilation gamma rays, $v$ is the velocity of the electron–positron pair at the time of annihilation, $m_e$ is the electron mass, and $c$ is the velocity of light. From this we find

$$
v = c \cos \beta / 2 \cos \Psi,
$$

where $\Psi$ is the angle between the $oZ$ axis and the vector $v$. It follows from the solution that if the electron–positron pair at the time of annihilation has velocity $v$ and one gamma ray reaches the point $K$ on the surface of the sphere, the second reaches the point $T$ formed by the intersection of the straight line passing through the end of the vector $v$ and the point $K$ with the surface of the sphere. An annihilation event is assumed to be detected if both gamma rays reach the detectors; this means the directions of emission of the gamma rays pass through the "shadows" of the detectors and the arrival of a gamma ray in the "shadow" signifies its arrival in the detector. The zone of nonzero detection $G(\alpha)$ is found as the locus of the points of the end of the vector $v$ through each of which one can describe a straight line intersecting both shadows of the detectors. For rectangular detectors of equal area, the zone $G(\alpha)$ is, to a high degree of accuracy, a rectangular parallelepiped for fixed angle $\alpha$. In a real experiment, the electron–positron pair velocity satisfies $v < c$, so that one gamma ray can with equal probability be emitted in any direction for fixed $v$. If the end of the vector $v$ is at the point $N$ of the zone $G(\alpha)$ and the gamma ray is emitted in the direction of the solid angle $\Omega_N$, reaching the hatched part of the shadow of detector I, the second gamma ray arrives in the hatched part of the shadow of detector II; for all other directions of emission of the gamma rays simultaneous arrival in the two detectors will not occur. The condition $v < c$ implies $\Omega_N = \Omega_N$, which simplifies the calculation of the detection probability for two-photon decay, $P_N$, for the point $N$ of the zone $G(\alpha)$:

$$
P_N = P(v, \alpha) = \Omega_N 4\pi = \Omega_N' 4\pi.
$$

The gamma-ray coincidence counting rate $I(\alpha)$ in this case has the form

$$
I(\alpha) = \text{const} \int \rho (v) P(v, \alpha) (dv),
$$

(1)