LOSS OF STABILITY OF A THIN-WALLED CYLINDRICAL SHELL UNDER PLASTIC ELONGATION

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We consider the loss of stability of plastic deformation manifesting itself in local necking with subsequent rupture. The critical strain corresponding to this form of stability is generally estimated by using engineering stability criteria. Tests for positive values of the added loads [9] and the work done by the added loads [11] are the most frequently used criteria. These criteria give good results only in special cases. In more general cases they are not confirmed by experiment. In addition, the engineering stability criteria for plastic deformation have not been adequately justified theoretically.

In forming shells from sheet material the critical strain is sometimes determined from the stability condition for plane elongation of the sheet, used as a local stability criterion [9]. In this case the error made in neglecting the effect of the curvature of the shell remains undetermined.

Using the principles presented in [5, 6] we investigate the stability of a cylindrical shell of isotropically hardened rigid plastic stretched by internal pressure. The flow curve of the material $\sigma_0 = \sigma_0\left(e_0\right)$ is supposed given. Here $\sigma = \sqrt{\frac{1}{2}} s_{ij} s_{ij}$ is the stress intensity; the $s_{ij}$ are the components of the stress deviator tensor; and $\epsilon_0$ is the accumulated strain intensity.

It is required to determine the smallest accumulated strain intensity for which an unstable form of equilibrium can exist, manifesting itself in local necking of the material.

Having fixed the strain $e_0 = e_0^0$ to be studied, we consider an additional sufficiently small range of strain $\delta e_0$. Here $\epsilon_0 = \sqrt{\frac{1}{2}} \delta e_{ij} \delta e_{ij}$ is the intensity of the strain increments $\delta e_{ij}$ measured from the strain $e_0^0$ under investigation.

By assuming that $e_0$ is small we can eliminate the geometric nonlinearity and use the Cauchy linear relations to determine the strain increments $\delta e_{ij}$ in terms of the displacement increments $u_i$:

$$\epsilon_{ij} = \frac{1}{2} \left( \frac{\partial}{\partial x_i} u_j + \frac{\partial}{\partial x_j} u_i \right).$$

In the range of strains under study the components of the stress-strain state satisfy the finite-difference form of the equations of state of flow theory

$$s_{ij} = \frac{2}{3} \frac{\sigma_0}{\epsilon_0} e_{ij},$$

the equilibrium equations

$$\frac{\partial}{\partial x_i} \sigma + \frac{\partial}{\partial x_j} s_{ij} = 0,$$

where $\sigma$ is the hydrostatic pressure; and the incompressibility condition

$$\epsilon_{ii} = 0.$$

If the strain $e_0^0$ is critical, then for a further strain the equilibrium state

$$s_{ij} = s_{ij}^0 + \delta s_{ij}; \quad \epsilon_{ij} = \epsilon_{ij}^0 + \delta \epsilon_{ij}; \quad u_i = u_i^0 + \delta u_i,$$

as well as the unperturbed states $s^0_{ij}$, $e^0_{ij}$, $u^0_1$ become possible. We call the infinitesimal increments $\delta s_{ij}$, $\delta e_{ij}$, and $\delta u_1$ additional components.

Assuming that the hardening is linear with a hardening index $\eta = d\sigma / d\epsilon$ in the range of strain $\delta \epsilon$, we have $\delta e_\epsilon = \eta \delta \epsilon_\epsilon$.

We solve the problem in cylindrical coordinates. We assume that the subcritical stress-strain state of the shell is plane and homogeneous [6]:

$$\sigma_\theta = \sigma_{r\theta} = s^0_{rz} = e^0_{rz} = 0, \sigma_\epsilon = \frac{\sqrt{3}}{2} \sigma_\phi = \sqrt{3} \epsilon_\phi.$$

Experiments show that the loss of stability of a strained cylindrical shell from internal pressure manifests itself in the formation of a neck rather drawn out in the axial direction $z$. Limiting ourselves to a consideration of this kind of loss of stability we take $\delta \epsilon_z = 0$ and assume that all derivatives with respect to $z$ are zero.

Under these assumptions we obtain from the linearization of system (1)-(4) a closed system of equations for the additional components:

$$\frac{\partial}{\partial r} \delta \sigma + \frac{\partial}{\partial \varphi} \delta \epsilon_r + \frac{1}{r} \left( \delta s_r - \delta s_\theta \right) + \frac{1}{r} \frac{\partial}{\partial \varphi} \delta s_{\varphi \varphi} = 0;$$

$$\frac{1}{r} \frac{\partial}{\partial \varphi} \delta \sigma + \frac{\partial}{\partial r} \delta \epsilon_\theta + \frac{\partial}{\partial \varphi} \delta s_{r\theta} + \frac{2}{r} \delta s_{\varphi \varphi} = 0;$$

$$\delta s_r = \frac{2}{3} \eta \delta \epsilon_r; \delta s_\theta = \frac{2}{3} \eta \delta \epsilon_\theta; \delta s_{r\theta} = \frac{2}{3} \frac{\sigma_0^2}{\sigma_\epsilon^2} \delta \epsilon_\epsilon;$$

$$\delta \epsilon_r + \delta \epsilon_\theta = 0;$$

$$\delta \epsilon_\varphi = -\frac{2}{3} \delta \epsilon_r; \delta s_{\varphi \varphi} = \frac{1}{r} \left( \delta u_r + \frac{\partial}{\partial \varphi} \delta u_\varphi \right); \delta \epsilon_r = \frac{\partial}{\partial r} \delta u_r;$$

$$(9) \quad \delta s_{\varphi \varphi} = \frac{1}{2} \left( \frac{\partial}{\partial r} \delta u_\varphi - \frac{1}{r} \frac{\partial}{\partial \varphi} \delta u_r + \frac{1}{r} \delta u_r \right).$$

This system of equations reduces to the following equation for $\delta u_r$:

$$\left[ r^2 \frac{d^2}{dr^2} + 6r \frac{d^2}{dr^2} + \left[ 5 - 2 \left( 1 - 2 \eta \frac{\epsilon_\phi}{\sigma_\epsilon} \frac{\partial^2}{\partial \phi^2} \right) \frac{\partial^2}{\partial \phi^2} \right] \frac{\partial^2}{\partial \phi^2} - \left[ 2 \left( 1 - 2 \eta \frac{\epsilon_\phi}{\sigma_\epsilon} \frac{\partial^2}{\partial \phi^2} + 1 \right) \frac{1}{r} \frac{\partial}{\partial r} + \frac{\partial^2}{\partial \phi^2} + 2 \frac{\partial^2}{\partial \phi^2} + 1 \right] \delta u_r = 0. \quad (10)$$

We seek a solution of Eq. (10) in the form

$$\delta u_r = R(r) \cos \lambda \varphi. \quad (11)$$

Here $\rho = r_1 / h$, where $r_1$ and $h$ are, respectively, the inside radius and thickness of the shell at the instant under investigation; and $\lambda$ is the neck parameter.

Substituting Eq. (11) into (10), we obtain Euler's differential equation

$$r^2 \frac{d^2 R}{dr^2} + 6r \frac{d^2 R}{dr^2} + \left[ 5 - 2 \left( 1 - 2 \eta \frac{\epsilon_\phi}{\sigma_\epsilon} \frac{\partial^2}{\partial \phi^2} \right) \frac{\partial^2}{\partial \phi^2} \right] \frac{\partial^2}{\partial \phi^2} - \left[ 2 \left( 1 - 2 \eta \frac{\epsilon_\phi}{\sigma_\epsilon} \frac{\partial^2}{\partial \phi^2} + 1 \right) \frac{1}{r} \frac{\partial}{\partial r} + \frac{\partial^2}{\partial \phi^2} + 2 \frac{\partial^2}{\partial \phi^2} + 1 \right] R \frac{dR}{dr} + (\lambda^2 \rho^2 - 1)^2 R = 0.$$

The introduction of a new variable $t = \rho \ln (r / r_1)$ transforms this equation to the form

$$R^{IV} - 2 \alpha R^" + \beta^2 R = 0. \quad (12)$$

where

$$\alpha = \lambda^2 \left( 1 - 2 \eta \frac{\epsilon_\phi}{\sigma_\epsilon} \frac{\partial^2}{\partial \phi^2} \right) + \frac{1}{\rho^2}; \quad \beta = \lambda^2 - \frac{1}{\rho^2}.$$

Since $\lambda > 1/\rho$, the general solution of Eq. (12) can be written in the form

$$R = (C_1 \cos \rightangle + C_2 \sin \rightangle) e^{\alpha t} + (C_3 \cos \rightangle + C_4 \sin \rightangle) e^{-\alpha t}. \quad (13)$$