The SOMS model and its application to Lake Neuchâtel

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ABSTRACT

A three-dimensional numerical circulation model (SOMS) based on primitive equations is described. The algorithm, by which Coriolis and vertical diffusion terms are treated implicitly while mass is still conserved exactly (algebraically), is discussed in detail. The model is applied to Lake Neuchâtel (Switzerland), to determine the general circulation under influence of the most prevailing wind.

1. Introduction

Mathematical computer models are currently being used with relevant observations to assess transport and dispersion of soluble and suspendable materials in lakes and oceans, originating from various sources. The model that is described in this paper is a three dimensional numerical circulation model based on primitive equations. Originally it was developed for ocean studies at Sandia National Laboratories in Albuquerque, and is referred to as the SOMS model (Sandia Ocean Modelling System). The model has advantageous stability characteristics and the operation-count per time step is relatively small compared with other 3D methods (see Dietrich, 1987). Therefore, a high resolution of the computational grid is possible. The model is applied to Lake Neuchâtel (Switzerland), to provide a first overview of the general circulation induced by the most prevailing wind.

This paper includes a description of the SOMS model and illuminates in particular the specific predictor-corrector scheme by which the surface pressure is determined and the part of the algorithm that treats Coriolis and vertical diffusion implicitly, while mass is still exactly conserved in the algebraic sense. Further, results of applying the model to Lake Neuchâtel under barotropic conditions are presented.
2. The continuum equations

2.1 Hypotheses and approximations

The Navier-Stokes (N-S) equations, a set of non-linear partial differential equations, together with an equation of state, an energy equation, and appropriate boundary and initial conditions govern Newtonian fluids (see Batchelor, 1967). The special nature of the Lake problem allows certain approximations that are used to simplify the governing equations. The resulting system is integrated using a fast computer such as a CRAY-2. Scaling estimates of the terms that appear in the N-S equations justify the following approximations (Pedlosky, 1987):

- The Boussinesq approximation: density variations are neglected in the horizontal momentum and mass conservation equations, but are taken into account when they are associated with buoyancy forces (in the vertical momentum equation). Density is assumed independent of pressure and varies only with temperature.
- The hydrostatic approximation: all terms in the vertical momentum equation are neglected except the buoyancy and pressure gradient terms.
- The $f$-plane approximation: a Cartesian coordinate system attached to the earth with constant Coriolis parameter.
- The rigid lid approximation (see below).
- A relatively simple turbulence closure scheme (see below).

Two categories of water motion can be distinguished, namely the gravitational response and the non-divergent response. The gravitational response is characterized by divergent motion, with a significant ratio of potential to kinetic energy. The timescale of this response is proportional to the length of the lake divided by the speed of the long gravity waves (the seiche period). It is in the order of minutes for Lake Neuchâtel. For the non-divergent response, however, most of the energy is kinetic and the time-scale is larger than the seiche period. The rigid lid (non-divergent) model which is applied here therefore simulates the transient, large scale circulation in the Lake.

When the equations are solved on a finite space-time domain, the molecular diffusion becomes negligible compared to turbulence effects at a scale smaller than the grid-size of this domain. Although variations at this scale are not resolved by the model, their effect on the resolved motion is important through non-linear interactions. The usual way to account for this effect is to decompose all variables into a mean value and a fluctuating value. Then, the resulting set of equations for the mean values, with the effect of the fluctuating part on the mean flow defined by additional model assumptions, has to be solved. Many levels of sophistication can be used to account for this turbulence part (Mellor and Yamada, 1982). Here, we use a rather popular and relatively simple method: the turbulence is represented in the mean flow equations by means of so-called turbulence viscosity coefficients, which are determined by a simple theoretical model (Pedlosky, 1987).