STABILITY OF JETS OF AN IDEAL PONDERABLE LIQUID

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The stability of jets of an ideal liquid was investigated in [1-4], where it was assumed that the undisturbed flow is parallel, and the velocity of the liquid in the jet is constant. In this paper we examine the stability of jets of ponderable liquids within the framework of linear theory, taking into account the effect of the surrounding medium, which is also assumed to be ideal. The ponderability of the liquid is manifested in the deviation of the jet boundaries from the parallel direction and the dependence of the velocity on the longitudinal coordinate. These features can be taken into account as, for instance, in the theory of stability of laminar boundary layers, where the flow is assumed to be quasi-parallel. In this case the dependence of the jet thickness and velocity in the jet on the longitudinal coordinate can be regarded as parametric. In this paper we examine a significantly nonparallel flow and, hence, for determination of the stability characteristics of a jet flow in this case we propose an asymptotic method.

1. Basic Equations. The basic equations have the form

\[ \frac{\partial^2 \Phi}{\partial t^2} + k \frac{\partial \Phi}{\partial r} + \frac{\partial^2 \Phi}{\partial r^2} = 0, \quad i = 1, 2, \]

\[ \frac{\partial \Phi}{\partial t} + \frac{u_i}{\rho_i} \frac{\partial \rho_i}{\partial x} + \frac{a_i^2 + v_i^2}{2} + gx = \text{const}_i, \]

where \( u_i = \partial \Phi_i / \partial x, v_i = \partial \Phi_i / \partial r \) are the projections of the velocity on the x and r axes, \( p_i \) is the pressure, \( \rho_i \) is the density; \( k = 0 \) for a plane jet, \( k = 1 \) for an axisymmetric jet; the subscript 1 relates to the flow parameters in the jet, and the subscript 2 relates to the surrounding medium. On the jet boundary the conditions

\[ v_i = \frac{\partial a_i}{\partial t} + u_i \frac{\partial a_i}{\partial x}, \quad p_1 - p_2 = \sigma (1/R + k/a), \]

\[ R = -\left[1 + \left(\frac{\partial a_i}{\partial x}\right)^2\right]^{1/2} \frac{\partial^2 a_i}{\partial x^2} \]

are fulfilled, where \( a \) is the radius \((k = 1)\) or halfwidth \((k = 0)\) of the jet; \( \sigma \) is the coefficient of surface tension.

Henceforth, we will deal with the problem in region 1 in the variables \( \xi = x/a_0, \tau = Ut/a_0, n = r/a_0 \), and in region 2 in the variables \( \xi, \tau, \) and \( N = (r - a)/a_0 \xi^m + k \), where \( a_0 \) is the linear scale; \( U \) is the velocity scale; \( m \) is a coefficient which will be determined below. Keeping within the framework of linear theory, we put the solutions of Eqs. (1.1) in the form

\[ \Phi_i = a_0 U (\psi_i + \xi \varphi_i), \quad p_i = \rho_i U^2 (P_i + p_i \delta), \quad a/a_0 = y_0 + \delta, \]

where the first terms on the right-hand sides correspond to undisturbed motion, and the second terms to disturbed motion.

In the new variables the equations for the disturbed motion and the boundary equations have the form (the velocity of the surrounding medium is zero)
2. Plane Jet \((k = 0)\). In this case the asymptotic form of the solution \((large \xi)\) of the undisturbed equations is fairly simple:

\[
\phi_1 = C \left( \frac{\xi^{3/2} - 3}{3} \right), \quad \phi_2 = \xi^{-1/2},
\]

\[
C = \begin{cases} \left( \frac{\alpha \phi_1}{U^2} \right)^{1/2}, & \frac{\rho^2}{\rho^1} \ll 1, \\ \left( \frac{\alpha \phi_2}{U^2} \right)^{1/2}, & \frac{\rho^2}{\rho^1} \gg 1. \end{cases}
\]

We will not consider the subsequent terms of the expansions, since their order lies outside the number of approximations considered in this paper. To determine the solutions of the above equations we put the first terms of the expansions of functions \(\phi_1, \phi_2, \) and \(\delta\) in the form

\[
\phi_1 = \phi_1^0 + \phi_1^1, \quad \phi_2 = \phi_2^0 + \phi_2^1, \quad \delta = \delta^0 + \delta^1 + \delta^2 + \ldots
\]

Substituting expressions (2.1) in the kinematic conditions (1.4), equating the orders of the first three terms in the first condition, and retaining the terms in the second, we obtain \([for the pressure we use the second equations of (1.2) and (1.3)]\)

\[
r = 1/2, \quad s = \beta = p + 3/2, \quad \alpha = \zeta = p + m.
\]

The value of \(m\) can be found from the condition that, in view of the boundedness of the potential in the external region at infinity, we must retain terms with second derivatives with