FAILURE OF SOLIDS UNDER STRAIN

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It is now generally accepted that failure is a process which begins virtually at the instant of load application to a specimen. Under a specific level of external forces this process is in its initial stage of a quasi-stationary character, and this makes it possible to consider it as an irreversible process and to apply to it methods of thermodynamics.

From the point of view of molecular structure the process of failure can be regarded as the development of structural irregularities, i.e., breakdown of the initial order as determined by internal forces between elements of a system and the conditions of its formation. The process is thus intimately related to changes of the system entropy. The same conclusion can be arrived at by considering the similarity of the melting and failure processes noted by Born [1, 2].

The condition of local failure is derived and a phenomenological description of failure is presented, making it possible to take into consideration the effect of various physicochemical factors. Examples of specific viscous failures are examined, and experimental and theoretical data are compared.

1. The analysis of similarity of failure and melting, and of the known empirical laws of melting [3], make possible the following assumption: A certain critical value of entropy density \( s^* \), which is a property of material,

\[
s(t^*) = s^* = s(0) + \Delta s^*
\]

(1.1)

corresponds to a local failure.

Here \( t^* \) is the instant of failure occurrence (in the following the asterisk denotes parameters related to the state of failure), \( s(0) \) is the density of entropy at the initial instant of time, and \( \Delta s^* \) is the critical increment of entropy density relative to the given value of \( s(0) \).

Equation (1.1) may be conveniently written in the form

\[
\int_0^{t^*} s'(\tau) d\tau = \Delta s^*
\]

(1.2)

The rate of density variation of entropy can be considered as the sum of the external flux \( s^*_e \) and of the internal source of entropy increase \( \eta \)

\[
s'(t) = s^*_e(t) + \eta(t)
\]

(1.3)

In conformity with the representation of irreversible processes in thermodynamics, we write the expression for the internal source of entropy increase in the form of bilinear thermodynamic fluxes and forces

\[
\eta = \sum_J J_a X_a
\]

(1.4)

As the thermodynamic fluxes \( J_\alpha \) and related forces \( X_\alpha \) we can choose, for example, the rate of chemical reactions and the chemical affinity, the stream of matter and the concentration gradient, the tensor of the irreversible strain rate and the stress tensor, etc.

From (1.2), (1.3), and (1.4) we obtain the condition of failure of the form...
Equation (1.5) takes into account the experimentally established fact of dependence of the failure process not only on the state of stress but, also, on various nonmechanical factors such as chemical transformations, diffusion within the material, temperature variation, and so on, i.e., failure is considered as a thermodynamic process and not as a (purely) mechanical one.

2. Let us consider the viscous failure of metal on the basis of the above premises. For simplicity we shall confine our analysis to the known Hoff's problem of failure of a metal rod of length $L_0$ and cross section $F_0$ in creep under constant tensile force $P$. The material is considered to be incompressible, i.e.,

$$\sigma(t) = \sigma_0 [1 - e^P(t)]$$

(2.1)

and subject to the law of creep flow

$$\dot{\varepsilon}^P(t) = B \sigma^m(t); \quad \sigma(t) = \frac{P}{F_0(t)}; \quad \sigma_0 = \frac{P}{F_0}; \quad e^P = \frac{t(t) - t_0}{t_0}$$

(2.2)

Here the dot in the superscript denotes a derivative with respect to time, and $B$ and $m$, which are properties of the material, are the coefficient and the exponent of creep (stress), respectively.

The experiments whose results are compared with theoretical data are, as a rule, carried out under isothermal conditions, which implies a nonzero external entropy flux $s_\infty$. However, for simplicity we assume $s^e = 0$, since for stabilized creep this assumption yields the same results as for the isothermal case, except that $\Delta s^*$ has a somewhat different value. The external entropy flux $s^e$ depends essentially on the rate of load application, and, as shown by experiments described in [4], a $4 \cdot 10^3$ times increase of the strain rate results only in a fourfold increase of the work of rupturing (the specimen). This shows that under conditions close to experimental [4], the variation of the value of critical entropy is only slightly affected, when $s^e$ is taken into account.

Hence the condition of failure (1.5) is written in the form

$$\int_0^t \sum_j J_{x_j} X_{x_j} d\tau = \Delta s^*$$

(2.3)

In this problem the dissipation of mechanical energy is the source of entropy increase. The change of the total strain work density is determined by the convolution of the strain rate tensor $T_{x\varepsilon}$ and of the stress tensor $T_\sigma$

$$\Lambda^* = T_{x\varepsilon} : T_\sigma$$

The strain rate tensor can be taken to be the sum of a perfectly elastic part $T_{x\varepsilon}^p$ and an irreversible part $T_{x\varepsilon}^s$:

$$T_{x\varepsilon}^* = T_{x\varepsilon}^p + T_{x\varepsilon}^s$$

(2.4)

Since a perfectly elastic deformation does not increase the medium entropy (see, e.g., [5]), the internal source of entropy, in the absence of any other factors, depends only on the rate of the irreversible strain.

For uniaxial stressing we assume

$$\eta(t) = \frac{1}{\theta} \dot{\varepsilon}^P(t) \sigma(t)$$

(2.5)

$(\theta$ is the absolute temperature)

For an isothermal process, in accordance with (2.3), we obtain

$$\Delta s^* = \frac{1}{\theta} \int_0^t \dot{\varepsilon}^P(t) \sigma(t) d\tau$$

(2.6)

Using relationship (2.1), we integrate the right-hand side of equality (2.6) and obtain