THEORY OF FILTRATION OF POLYDISPERSE AEROSOLS FROM A STEADY STREAM

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The process by which aerosols are filtered in various engineering installations is something that is determined both by the properties of the aerosols themselves and by the independent properties of the filter. An aerosol is distinguished by the dimensions and form of the particles as well as by the particle count concentration, while the main property of the filter is expressed as the degree of purification from the aerosol that it gives or by the amount of the aerosol that gets through for a given amount of air passed. For an aerosol with the simplest particle form the most important thing is the dimensions of the particles and how their concentration is distributed. The simplest system would be an ideal aerosol where the particles are all the same size, but using and absorbing aerosols with different size particles will always be of particular importance for the following reasons: 1) under the actual conditions in which filtration installations are used we can only think of aerosols with different size particles, while aerosols with only one size particles can only be produced under special laboratory conditions, 2) in evaluating and testing filters we have to have definite particle dimensions in mind if we are going to make a correct and rigid evaluation of the filter, and so it is impossible to get along without making some preliminary experiments on the absorption of aerosols with different particle sizes, 3) in solving the underlying problems of the mechanism of the filtration process, using aerosols with different size particles experimentally frees the investigator from the necessity of having a selection of different aerosols each having its own particle size and having to worry about the strict reproducibility of his experimental conditions [1].

The ideas presented below deal with filtration of aerosols with different size particles under the simplest possible conditions, where we have a steady stream and the precipitation of each fraction goes independently of the precipitation of the others. The results so obtained are convenient for analyzing the processes taking place in the following actual installations: 1) a filter made of fibrous material as well as a sand filter, 2) a tube or plate type diffusion battery (as used for analyzing aerosols) [2], 3) any engineering installation for absorbing aerosols on the mechanical principle (bag filters, cyclones, etc.), 4) individual obstacles intended to precipitate aerosols from the stream (cylinders, spheres, plates, corrugated surfaces, etc.).

The simplest parameters of the majority of these installations are the partial escape coefficient and the precipitation efficiency (for single obstacles). For a given installation (filter) these quantities are constant for a fixed airflow, depending only on the aerosol particle dimensions but not on the distribution in dimensions. Accordingly they may be considered to be invariant quantities in the filtration process. Let us stop to consider first of all the first three problems of filtration where the principal parameter is the escape coefficient. If, as is usually done, we divide the aerosol into fractions with finite ranges of particle dimensions (radii) $\Delta r$ in each fraction, what we call the partial escape coefficient $K(r_i)$ will be the ratio of the particle concentration of the $i$-th fraction, $\Delta N_i$, after filtration to the initial concentration of the same fraction, $\Delta N_i$, in the aerosol before filtration, i.e., the ratio

$$K(r_i) = \frac{\Delta N_i}{\Delta N_i}.$$  

The particle concentration in the original aerosol for the fraction with particle dimensions from $r_i$ to $r_i + \Delta r$ may be expressed as

$$\Delta N_i = N_{\phi\psi}(r_i) \Delta r.$$
where \( N_0 \) is the total number of all particles of all dimensions per unit volume of aerosol, and \( \varphi(t) \) is the value in the density distribution for the \( i \)-th fraction. In a similar way, the number of particles in the fraction from \( r_i \) to \( r_i + \Delta r \) for the aerosol that has escaped is

\[
\Delta n_i = n_0 \int f(r_i) \Delta r,
\]

where \( n_0 \) is again the total number of particles per unit volume in what has escaped and \( f(r_i) \) is the corresponding density in the distribution. The partial escape coefficient for the \( i \)-th fraction is by definition

\[
K(r_i) = \frac{\Delta n_i}{\Delta N_i} = \frac{n_0 f(r_i)}{N_0 \varphi(r_i)}.
\]

Obviously, the mean escape coefficient (or mathematical escape expectancy \( K \)) is equal to

\[
\bar{K} = \frac{n_0}{N_0} = \sum K(r_i) \varphi(r_i) \Delta r.
\]

So that

\[
K(r_i) = \bar{K} \frac{f(r_i)}{\varphi(r_i)}.
\]

Here, within each fraction \( f(r_i) \) and \( \varphi(r_i) \) are constant, and the distribution is given by a step diagram. Going to the limit as \( \Delta r \to 0 \), we will have for the continuous functions \( K(r) \), \( \varphi(r) \), and \( f(r) \) at constant stream velocity and fixed parameters of the installation the general expression \( K(r) = \frac{dn}{dN} \). If, as usual, we set

\[
dW = \frac{dN}{N_0} = \varphi(r) dr; \quad dw = \frac{dn}{n_0} = f(r) dr
\]

with the conditions

\[
\int \varphi(r) dr = 1; \quad \int f(r) dr = 1,
\]

to determine \( K(r) \) we have

\[
K(r) = \frac{n_0 f(r)}{N_0 \varphi(r)}.
\]

Since

\[
n_0 = \int K(r) dN,
\]

we have

\[
\bar{K} = \frac{n_0}{N_0} = \int K(r) \frac{dN}{N_0} = \int K(r) dW
\]

and, accordingly, \( \bar{K} \) again becomes the mean escape coefficient or mathematical escape expectancy \( K \). Obviously

\[
\bar{K} = \int K(r) \cdot \varphi(r) dr.
\]

Hence from (3) we find

\[
K(r) = \bar{K} \frac{f(r)}{\varphi(r)} = \bar{K} \cdot F(r).
\]

The initial distribution \( \varphi(t) \) is in no way dependent on the filter, while the distribution \( f(t) \) on the other side of the filter is a function of \( K \) and \( \varphi \), and thus depends on the filter and on the initial distribution, and, since both quantities are related to \( r \), it is a function of the particle dimensions. From what has been said we can write \( f(t) = f(K, \varphi) \).

Going from the argument \( r \) to the independent variables \( K \) and \( \varphi \), we find that \( r = u(\varphi) \) or \( dr = u'(\varphi) d\varphi \), and so

\[
\bar{K} = \int K \cdot \varphi \cdot u'(\varphi) d\varphi.
\]

Then (5) goes over into the relation

\[
K = \frac{\int K \cdot \varphi}{\varphi}.
\]