DYNAMICS OF THE BEHAVIOR OF A GAS-BUBBLE NUCLEUS IN A HETEROPHASE MEDIUM

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The behavior of the nucleus of a gas bubble in a heterophase medium is an important problem in the investigation of the evolution of gas-shrinkage porosity in alloys crystallized in a certain temperature interval [1-4] and in a study of dynamical and mass-transfer phenomena in gas-liquid systems moving through a porous disperse media [5-8]. The general solution of this type of problem under the conditions of inhomogeneity of the temperature and concentration fields and in the presence of convection motions of the liquid phase poses a complicated mathematical problem. We therefore confine the ensuing discussion to a simplified mathematical model of the growth of the nucleus of a gas bubble in a homogeneous quasiequilibrium zone of a binary alloy [9], generalizing the solution to the case of the growth of a gas bubble in an isothermal liquid-saturated porous disperse medium.

We consider the crystallization of a binary alloy containing dissolved gas. We assume that the volume occupied by the alloys is small enough for the internal thermal resistance of the substance to be neglected in comparison with the external thermal resistance and for the crystallization of the alloy to be regarded as a volume process. We neglect shrinkage effects in crystallization, assuming that the nucleation of a bubble is associated with the displacement of dissolved gaseous component by the growing solid phase, while the motion of the melt is elicited by the variation of the gas-bubble radius due to gas diffusion from the intercrystalline liquid. We also assume that the vapor density in the bubble interior is negligible in comparison with the density of the gas, the distance between the centers of the bubbles is much larger than the characteristic diameter of the dendritic (structural) cell, and the diameter of the bubble itself is so small that the convective diffusion of the gas toward the bubble surface as a result of its ascension can be disregarded. The equations of continuity and momentum transfer have the following form in a spherical coordinate system attached to the center of the bubble [9]:

\[ \frac{\partial}{\partial r} (r^2f_\theta u) = 0; \]
\[ \rho \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial r} \right) = -\frac{\partial p}{\partial r} - \frac{\mu}{K_p(f)} + \frac{\mu}{r^3} \left[ \frac{\partial}{\partial r} \left( r^2 \frac{\partial u}{\partial r} \right) - 2u \right], \]

where \( u \) is the velocity of the liquid, \( f_\theta \) is the cross section of the liquid phase (porosity), \( p \) is the pressure in the liquid, \( K_p(f) \) is the permeability of the heterogeneous zone, \( \rho \) is the density of the liquid, \( \mu \) is the dynamic viscosity of the liquid, and \( r \) is the radial coordinate. Equations (1) and (2) must be integrated subject to the boundary conditions on the surface of the bubble (\( r = r_\text{p} \)).

Here \( r_p \) is the rate of change of the bubble radius, and \( p_g \) is the gas pressure in the bubble interior. We assume that the concentrations of the alloying component (\( C_1 \)) and the gaseous component (\( C_2 \)) far from the bubble \((r \to \infty)\) are related to the cross section \( f_p \) of the liquid phase by Scheil's rule [1]:

\[
C_i = C_{i0}/f_p^{1-k_i},
\]

where \( k_i \) is the distribution coefficient of the \( i \)-th component. Neglecting the influence of dissolved gas, we specify the liquidus temperature in the form of a linear function of the concentration \( C_1 \):

\[
T_s = T_A - \beta_{10} C_1.
\]

It follows from the quasiequilibrium condition [1, 2] that

\[
T = T_A - \beta_{10} C_1.
\]

We obtain from Eq. (5) with \( i = 1 \) and from Eq. (6)

\[
f_i = \left( \frac{T_A - T_{10}}{T_A - T} \right)^{1/(1-k_i)},
\]

where \( T_{10} = T_A - \beta_{10} C_{10} \).

We assume for definiteness that the cooling rate of the melt \( v_T = \partial T/\partial t = \text{const} \), so that

\[
T = T_{10} - v_T t.
\]

Then from Eq. (7) with allowance for (8) we have

\[
f_i(t) = [1 + (v_T/\Delta T) t]^{-1/(1-k_i)}, \quad \Delta T_0 = T_A - T_{10}.
\]

Integrating Eqs. (1) and (2) subject to the boundary conditions (3) and (4), we obtain

\[
\begin{align*}
\frac{d}{dt} r_p &= \frac{u}{r_p}; \\
r_p \frac{d^2 r_p}{dr^2} + \frac{3}{2} \frac{r_p}{r_p} + \nu \left[ \frac{f_p}{K_p(f_i)} + \frac{4}{r_p} \right] \frac{d}{dr} \left( r^2 \frac{dC_2}{dr} \right) + \frac{k \nu_T/\Delta T_0}{1 + \nu_T t/\Delta T_0} C_2, \quad k^* = \frac{1-k_2}{1-k_1};
\end{align*}
\]

Here \( p_\infty \) is the pressure far from the bubble, which is equal to the sum of the gas pressure above the surface of the melt and the metallostatic pressure at the level of the bubble, and \( \nu = \mu/\rho \).

The distribution of the concentration of the gaseous components in the liquid surrounding the bubble is given by the convective diffusion equation [9], which we write with allowance for Eqs. (9) and (10) in the form

\[
\frac{\partial C_2}{\partial t} + \frac{r_p^2 \partial C_2}{r^2} + \frac{D_2 \partial}{dr} \left( r^2 \frac{\partial C_2}{dr} \right) + \frac{k \nu_T/\Delta T_0}{1 + \nu_T t/\Delta T_0} C_2, \quad k^* = \frac{1-k_2}{1-k_1};
\]

We augment this equation with the initial and boundary conditions

\[
\begin{align*}
C_2 \big|_{t=t_p} &= C_{20}/f_i^{(1-k_2)}; \\
C_2 \big|_{r=\infty} &= C_{20} (1 + \nu_T t/\Delta T_0)^{k^*},
\end{align*}
\]

where \( f_{LP} \) is the cross section of the liquid phase at the instant \( t_p \) of nucleation of the bubble and is determined from the condition