NUMERICAL SIMULATION OF THE INJECTION OF A POWERFUL ELECTRON BEAM INTO A VACUUM CHAMBER HAVING A STRONG MAGNETIC FIELD

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The problem of beam transporting in a vacuum ranks high among the numerous problems associated with the use of powerful relativistic electron beams (REB) in physics experiments. Here it is analytically possible to consider only several of the simplest special cases; the majority of the situations require the use of numerical methods. The numerical procedures existing now are sufficiently effective for time-independent problems: they permit obtaining very high accuracy with totally acceptable expenditures of machine time [3-5]. Time-dependent problems are far more laborious and usually lie at the limit of possibilities of contemporary computational technique [6]. On the other hand, due to the comparatively short duration of powerful REB, time dependence often proves to be fundamentally important for them; in general it is necessary to take into account, along with the time dependence of the particle motion, the effects of retardation for the electric and magnetic fields. Since the corresponding calculations are very cumbersome, it is advisable to consider as the first step a situation in which the particle trajectories possibly appear more simply. This occurs upon the motion of the beam in a strong external magnetic field, when one can treat the particles as "strung" onto the force lines. This is precisely the case which is discussed in this paper. We note that the indicated formulation of the problem is sufficiently realistic, since in many experimental setups a strong field is created which predetermines the appearance of the electron trajectories.

1. The experimental scheme under discussion for transporting an REB is of the following form. The beam enters a cylindrical vacuum chamber through the anode foil, which is located at the end of the cylinder. Passing through the drift space, the beam is incident on the other end, which is the collector. The chamber walls, the foil, and the collector are at zero potential. An external magnetic field is uniform in the direction along the chamber axis (it is understood that the entire system possesses axial symmetry).

We will represent the beam in the drift chamber in the form of a set of tubular beams, each of which has a negligibly small thickness and can be taken into account as a boundary condition in the solution of Maxwell's equations. By properly specifying the number of tubular beams, their currents, and their radii, one can actually model an arbitrary distribution of injected current. For simplicity's sake we will formulate the problem for the case in which only one current tube occurs in all.

In cylindrical coordinates $(r, z)$ the drift space has a triangular form $0 \leq r \leq R$, $0 \leq z \leq L$. The current tube divides this region into two subregions: I $(0 \leq r \leq r_b)$ and II $(r_b \leq r \leq R)$.

In view of the axial symmetry of the problem, the system of Maxwell's equations is split into two independent systems (for TE- and TM-waves). The beam affects only the TM-waves in which the quantities $E_r$, $E_z$, and $H_q$ are different from zero:

$$\frac{\partial E_r}{\partial t} = -c \frac{\partial H_q}{\partial z}, \quad (1.1)$$
$$\frac{\partial E_z}{\partial t} = \frac{c}{r} \frac{\partial}{\partial r} (r H_q), \quad (1.2)$$
$$\frac{\partial H_q}{\partial t} = c \left( \frac{\partial E_z}{\partial r} - \frac{\partial E_r}{\partial z} \right). \quad (1.3)$$

The boundary conditions on the walls of the drift chamber are of the form

$$E_z = 0 \quad \text{at} \quad r = R, \quad H_q = E_r = 0 \quad \text{at} \quad r = 0,$$
$$E_r = 0 \quad \text{at} \quad z = 0 \quad \text{and} \quad z = L$$

It is necessary to supplement these formulas by matching the fields at \( r = r_b \) (on the beam):

\[
E_r^{\text{II}} - E_r^{\text{I}} = 4\pi \sigma, \quad H_\theta^{\text{II}} - H_\theta^{\text{I}} = \frac{4\pi}{c} j.
\]

(1.4)

Here \( \sigma(z, t) \) and \( j(z, t) \) are the surface charge and current densities. The function \( E_z \) is continuous at \( r = r_b \) \( (E_z^{\text{II}} = E_z^{\text{I}}) \), and its derivative with respect to the radius undergoes a discontinuity:

\[
\frac{\partial E_z^{\text{II}}}{\partial r} - \frac{\partial E_z^{\text{I}}}{\partial r} = \frac{4\pi}{c} j, \quad \frac{\partial \sigma}{\partial r} = \frac{4\pi}{c} j.
\]

(1.5)

We determine the variation of the charge density and current by proceeding from the equations of motion of the electrons:

\[
\int \frac{m_e v^3}{1 - v^2/c^2} \frac{dx}{v} = -eE_z(r_b, z, t), \quad \frac{dz}{dt} = v.
\]

The total energy of the system \( W \) is comprised of the energy of the electromagnetic field

\[
W_1 = \int \frac{1}{4} \left( E_r^2 + E_z^2 + H_\phi^2 \right) \, dr\, dz
\]

and the kinetic energy of the electrons

\[
W_2 = -\frac{2\pi m_e c^2}{e} r_b \int (v)(z) dz,
\]

where \( \gamma = (1 - v^2/c^2)^{-1/2} \). The variation of \( W \) is related solely to the fact that the electrons cross the boundaries of the region \( 0 \leq z \leq z_L \):

\[
\frac{dW}{dt} = \frac{2\pi m_e c^2}{e} r_b [(\nu^2)_{z=L} - (\nu^2)_{z=0}].
\]

(1.6)

Equation (1.6) will be used in the following as a control on the computational accuracy.

2. A scheme with overstepping is used for the numerical solution of Eqs. (1.1)-(1.3). For this purpose the functions \( E_r, E_z, \) and \( H_\phi \) on the beam are determined from Maxwell's equations with the boundary conditions (1.4) and (1.5) taken into account. The method of particles in cells is used to solve the equations of motion. The surface charge and current density necessary in conditions (1.4) and (1.5) are determined from the positions and velocities. The indicated algorithm is outlined in detail in [7].

The initial conditions were specified as follows in all the calculated versions. At the initial instant of time the functions \( E_r, E_z, \) and \( H_\phi \) are equal to zero and there are no particles in the drift space. Then at each time step an identical number of particles enters the drift space through the boundary \( z = 0 \) in order to provide a constant injected current.

3. Systematic calculations were performed in order to check the accuracy of this algorithm. One of them was the following. The injected current is increased very slowly from zero to some value \( I_0 \). The system arrived at a state differing little from the steady state which exists for a given value of the current. The parameters of the problem were as follows:

\[
I_0 = 0.031, \quad \nu = 0.9c, \quad r_b = 0.4R, \quad L = 5R, \quad h_r = h_z = 0.1R, \quad \tau = 0.01R/c, \quad \text{and} \quad 2000 \text{ particles}.
\]

Here and later the current is measured in units of the critical vacuum current \( I_\star \) for a tubular beam in an infinitely long system [8]

\[
I_\star = \frac{\nu^3 I_0}{\ln (R/r_b)} (\gamma^2 - 1)^{3/2}.
\]

(3.1)

where \( \gamma = (1 - \nu^2/c^2)^{-1/2} \). The injected current increases linearly in a time \( t = 22R/c \), and at the time \( t = 30R/c \) the deviation of all the quantities \( (I, \nu, \sigma) \) at the middle of the beam from the values of the steady solution did not exceed 0.05%. The deviation of the total energy from the values of the steady solution was 0.052% at this very same time.

The accuracy was controlled in the computational process by checking fulfillment of the law of conservation of energy. For this purpose the energy \( W_0(t) \) which the system should have up to time \( t \) is calculated. It is calculated by integrating Eq. (1.6) over the time. On the other hand, the energy can be calculated as the sum of the quantities \( W_1 \) and \( W_2 \) (see Sec. 1). The discrepancy between the energies calculated by these two methods is a measure of the computational accuracy.