1. Introduction. Electrical discharges in liquids have been examined repeatedly, but up to now there has been no satisfactory model that can be used with given discharge-circuit parameters \((C, L, U_0, R_0, l)\) to calculate the hydrodynamic flow. The basic reason is the inadequate development of the theory of dense nonideal low-temperature plasma in channels with inhomogeneity over the cross section and variable numbers of particles. Progress in this area is held back by the restricted scope for experimental investigation, particularly as regards the voltage and current in the discharge, the channel geometry, and sometimes the surface temperature.

Important information is provided by pressure determination in the channel. At present, the only method is by the interpretation of optical measurements on the wall path. The recording system of Fig. 1 enables one to record the size of the channel in transmitted light together with shock-wave propagation. A characteristic SFR recording (Fig. 2) shows the primary wave propagating at acoustic speed. The main shock wave is produced at a certain distance from the channel, and up to this time there is simply a nonstationary high-pressure compression wave.

The pressures arising in discharge channels have frequently been measured [1-3] via the assumption of a constant expansion velocity in the first quarter period. Then there is a self-modeling solution to the one-dimensional gas-dynamic problem [4-6]. For expansion speeds \(r_c \ll c_0\), the approximation of linear acoustics has been used [7, 8], which incorporates the velocity variation. Recently, attempts have been made [9-11] to calculate the liquid flows around sparks by numerical methods.

2. Formulation. A numerical method was used to construct the flow picture from the kinematics of the expansion. Usually, the density of a plasma is less by two orders of magnitude than the density of water, and therefore the possible error in determining the channel radius arising from the flow of material through the boundary is \(\Delta r/r < 0.01\). Therefore, the piston permeability may be neglected from the viewpoint of the hydrodynamic flow.

We calculated the one-dimensional flow of a liquid initially at rest on the expansion of an impermeable cylindrical piston. The Neumann–Richtmyer [12] artificial-viscosity method was used to calculate the flow with shock waves by means of a cross-type difference scheme. The equation of state for water is \(p = A(p/p_0)^m - B\) up to 20 kbar and was approximated by the expression

\[
p = 0.001 + x(21.77 + x(66.95 + x(11.92 + x(57 + 27 x)))
\]

where \(x = p/p_0 - 1\), which reduces the run time considerably. See [13] for details of the calculation method and approximation of the equation of state. The Walker–Sternberg [14] equation of state was used for high pressure.

Reliable determination of the time dependence of the pressure in the channel requires correction for the distortion of the recorded size arising from the change in the refractive index of water with pressure, and also the error in measuring the expansion of the channel due to the small size at the initial stage \((t \ll 1 \mu\text{sec})\). We consider the effects of each of these factors.

3. Optical Distortions Due to Refractive-Index Change in the Water. The recording system (Fig. 1) shows that the following formula applies [15] for the visible shadow radius of the channel:

\[
r_{\bullet} = r_c \left( \frac{n}{n_0} \frac{\Delta n}{n_0} + \frac{n}{n_0} \frac{r_1}{c} + \frac{\Delta n}{n_0} \frac{r_1}{c} \right)
\]

where \(n_0\) is the refractive index of water under normal conditions, \(n = n(p)\) is the refractive index at a given pressure and normal temperature, \(\Delta n\) is the change in refractive index at a given pressure due to heating (the values of \(n\) and \(\Delta n\) are taken for \(r = r_0\)), and \(x_1 = y_0 - r_c\) is the minimum distance from the boundary of the channel to the path of the ray that indicates the boundary in shadow recording. All rays passing near the boundary of the channel are highly deviated on account of the gradients in the refractive index due to heating, and they fall outside the aperture of the recording instruments \((\alpha > \alpha_{\bullet})\). Data on the absorption spectrum of water [16-18] were used in deriving upper bounds to the heating of the....
liquid arising from the absorption of ultraviolet radiation and from conduction. The thickness of the heated layer was \( \Delta x \ll r_c \) in discharges with channel temperatures up to 50,000° within the time range of interest. Then the beam deviation due to the refractive-index gradients arising from heating is

\[
\frac{|\Delta n|}{n} = 1 - \cos \left( \frac{\alpha_\tau}{2} \right) \approx \frac{\alpha_\tau^2}{8},
\]

since \( \alpha_\tau \approx \alpha_T \).

In choosing the maximum possible angle of illumination \( \alpha_* \) in the range \( 10^\circ \lesssim \alpha_* \lesssim 20^\circ \) for the ray detecting the boundary we obey the condition \( |\Delta n|/n \approx 0.02 \); in that case the ray passes in a comparatively cool region \( \Delta T \ll 1000^\circ \), and therefore the density dependence of the refractive index can be derived from the following expression [19]:

\[
n = 1 + 0.334\rho.
\]

(3.2)

The \( \rho(p, T) \) dependence from [20-24] can be used to show that \( |\Delta n|/n \approx 0.02 \) requires heating by about 100°. This corresponds to \( x_1 < 0.01 \) \( r_c \) from estimates of the heating, and with the optimal choice of \( \alpha_* \) it follows from (3.1) that with an error less than 2%:

\[
r_\tau = r_c n(p)/n_0.
\]

(3.3)

Therefore, the measured radius is greater than the true radius arising from the optical distortion caused by the elevated pressure near the channel.

If the boundary condition for the piston is put as \( r_p(t) = kr_\tau(t) \), where \( k = n_0/(1 + 0.334\rho) \), and \( \rho \) is the density of the water near the boundary of the channel outside the heating zone, then the optical distortions will balance out in calculating the flow. The boundary condition was approximated via the inexplicit scheme \( r_{p+1} = k_{1/2}r_{p-1} \). The system of inexplicit equations for the first cell in each time layer was solved by iteration. The channel expansion indicated by the set of experimental points \( r_i(t_i) \) was approximated by cubic spline interpolation. This provided continuity in the first and second derivatives at the interpolation nodes, which is necessary in calculating the hydrodynamic flows when there is gradual energy deposition in the discharge channel.