THE PROCESS OF SPALL FRACTURE

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Many investigations of spall phenomena after the emergence of the compression pulse to the free surface of a specimen show that the strength value realized in spalling depends on the characteristic time of action of the load. A number of studies [1-4] propose discrete criteria for spall fracture which determine the possibility of failure in terms of the value of the tensile stress and the time during which it acts at a particular cross section of the specimen. However, on the one hand, the load at any cross section may, in general, vary arbitrarily, and on the other hand, the failure process itself leads to a drop in the tensile stress, which makes the actual application of the discrete spall criteria difficult. The authors of [5-7] discuss the possibility of introducing a continuous measure of failure into the spall criterion; such a measure may be the dimensions and number of the cracks in the specimen, the residual strength of a half-ruptured specimen, etc. Experimental information on the failure process can be obtained from a metallographic analysis of preserved specimens [5, 6], or from experiments on the continuous recording of the velocity of the free surface of the specimen when a compression pulse and a “spall” pulse emerge onto it [8-11]. It is impossible at the present time to obtain continuous quantitative information directly from the failure zone.

In the present article we consider the effect of the kinetics of failure on the gas dynamics of a wave process. In the gas-dynamic analysis of a phenomenon, it is most convenient to use the specific volume of a crack, \( v_c \), as the measure of the failure. The shear strength of the medium will be disregarded in what follows. The rate of growth of the cracks (or pores), as can be deduced from general considerations [6, 7], is determined by the value of the negative pressure \( p \) acting on the material and by the degree of failure achieved, \( v_c \):

\[
\frac{dv_c}{dt} = f(p, v_c).
\]

Barbee et al. [6] have proposed specific expressions for the failure kinetics of (1), which are based on a model of exponential generation and ductile growth of the cracks.

In order to see what kind of information concerning the effect of continuous failure on the gas dynamics of the process can be obtained in general form, we shall follow the change of state of a substance along the characteristics in a linear material, i.e., in a material whose equation of state has the form

\[
\frac{\rho^2}{\rho_0^2} \left( \frac{\partial v}{\partial p} \right)_{v_0} = \text{const} = n^2 = \text{const},
\]

where \( \rho, \rho_0 \) are the instantaneous and initial values of the density of the substance. The specific volume of the failed medium, \( v \), consists of the volume of the solid material, \( v_{\text{sol}} \), and the volume of the cracks, \( v_c \):

\[
v = v_{\text{sol}} + v_c.
\]

The equations of motion and continuity, taking account of (1)-(3), for a one-dimensional case have in Lagrangian coordinates the form

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\[
\rho \frac{\partial u}{\partial t} + \frac{\partial p}{\partial h} = 0, \quad \frac{\partial p}{\partial t} + \frac{1}{\rho a} \frac{\partial u}{\partial h} - \rho a^2 \frac{\partial v_{cr}}{\partial t} = 0.
\]

(4)

where \( u \) is the mass velocity of the substance, \( h \) is a Lagrangian coordinate.

To determine the characteristic directions, we express the derivatives with respect to time in the system of equations (1)-(4), which describe the motion of the failed medium in terms of the derivatives \( \frac{d}{dt} \) in the direction \( \frac{dh}{dt} = \lambda \):

\[
\frac{1}{\rho} \frac{\partial p}{\partial h} - \lambda \frac{\partial u}{\partial h} = -\frac{d \lambda}{dt},
\]

\[
\lambda \frac{\partial p}{\partial h} - \rho a^2 \frac{\partial u}{\partial h} - \rho a \frac{\partial v_{cr}}{\partial t} = \frac{dp}{dt} - \rho a^2 \frac{\partial v_{cr}}{\partial t},
\]

(5)

\[
\lambda \frac{\partial v_{cr}}{\partial h} = \frac{dv_{cr}}{dt} - f(p, v_{cr}).
\]

By definition, the selected direction \( \lambda \) will be characteristic in the case when the determinant of the system (5) vanishes. From this we determine \( \lambda = \pm a, \lambda = 0 \), and consequently the characteristics in the case under consideration, as in a nonrelaxing medium, are straight lines with inclination \( \frac{dh}{dt} = \pm a \) and a particle trajectory \( h = \text{const} \).

The derivatives of the mass velocity and the motions along the characteristics \( C_+ \), taking (2)-(4) into consideration, have the form

\[
\frac{d u}{d t} \bigg|_{C_+} = \frac{\partial u}{\partial t} + \frac{1}{\rho a} \frac{\partial p}{\partial t} + \rho a \frac{\partial v_{cr}}{\partial t},
\]

\[
\frac{d p}{d t} \bigg|_{C_+} = \rho a \frac{\partial u}{\partial t} = -\rho a \frac{\partial v_{cr}}{\partial t} \bigg|_{C_+} + \rho a v_{cr}.
\]

(6)

Analogously, along the characteristics \( C_- \)

\[
\frac{d p}{d t} \bigg|_{C_-} = \rho a \frac{d u}{d t} \bigg|_{C_-} + \rho a v_{cr}
\]

(7)

From (6), (7) it can be seen that when \( \dot{v}_{cr} > 0 \), the trajectories of the change of state along the characteristics in the coordinates \( p, u \) deviate from the straight lines \( p = \pm \rho a u + \text{const} \), defined by the Riemann invariants, in the direction of higher pressure.

We consider the variation of the amplitude of a tension wave after reflection from a free surface of a triangular compression pulse being propagated in a positive direction. We shall indicate by + the states immediately in front of the tension jump, and by — the states immediately behind the jump. For a rarefaction jump being propagated in a negative direction, the Rankine—Hugoniot condition is satisfied:

\[
p^+ - p^- = -\rho a (u^- - u^+).
\]

(8)

Bearing in mind that the reflected wave is superimposed on the incident simple compression wave, for which \( \dot{v}_T = 0 \) and \( dp = \rho a du \), we find from (7), (8) that

\[
\left. \frac{dp}{dt} \right|_{C_-} = 2\left. \frac{dp}{dt} \right|_{C_+} - \rho a \frac{d u}{d t},
\]

\[
\dot{v}_{cr} = 2p_o + \frac{1}{2} \rho a^2 \dot{v}_{cr}.
\]

(9)

where \( p_o = \frac{1}{2} \left. \frac{dp}{dt} \right|_{C_+} \bigg|_{h_0} \) is the rate of change of the pressure in the incident pulse at the moment when the tension jump approaches the given particle; \( \dot{v}_{cr} \) is the initial rate of failure immediately behind the jump. According to (9), when \( \dot{v}_{cr} > 0 \), the growth in the amplitude of the tension wave takes place more slowly than in the case of an unfailed medium. This deduction must be taken into consideration when we determine the spall strength of materials. As an example, we shall consider the case when the rate of failure is a linear function of the pressure:

\[
\dot{v}_{cr}(p) = -2A p^+ / p_o a^2.
\]

For an incident triangular pulse \( p_o = \text{const} \) we can integrate (9) to obtain an expression for the amplitude of the tension wave

\[
p^- = 2p_o [1 - \exp(-At)] / A = 2p_o \left[ 1 - \exp \left( A \frac{h - h_o}{a} \right) \right] A.
\]

(10)

where \( h_o \) is the coordinate of the free surface of the specimen. According to (9), (10), the amplitude of the tension wave asymptotically approaches a value determined by the condition

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802