PROPAGATION OF PERTURBATIONS IN A LIQUID CONTAINING GAS BUBBLES

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One of the models of a bubble-containing medium (Iordanskii's system of equations of motion), based on liquid motion "averaged" on the assumption that bubble pulsation conforms to the Lamb equation, is investigated.

Solution of Iordanskii's linearized system gives a relationship between the phase velocity of sound and the plane-wave frequency. An evaluation of this relationship for a particular bubble size distribution agrees with known experimental results.

If the liquid component of the medium is incompressible, Iordanskii's system for bubbles of one kind reduces approximately to a system of two second-order partial differential equations for the pressure and concentration of gas in the medium. A solution of this system is found. For particular relative values of the parameters of the medium (length, gas concentration, and bubble size) the processes of perturbation propagation in bubble-containing media are similar. The similarity criterion is found from the system solution and is confirmed experimentally.

All the known theoretical works devoted to the considered question can be divided into two approaches, each of which has its own model of a bubble-containing medium. Both approaches are based on the "averaged" motion of the liquid containing gas bubbles. The difference is that in one approach [1, 2] the pressure in the gas bubble is always equal to the pressure in the liquid, and in the other [3] the pulsation of the bubbles is given by the Lamb equation. The calculations presented below are based on Iordanskii's equations [3].

These equations in the unidimensional case have the form

\[
\frac{\partial \rho}{\partial t} + \frac{\partial \rho u}{\partial x} = 0, \quad \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + \frac{1}{\rho} \frac{\partial p}{\partial x} = 0, \quad p = \left( \rho_0 + \frac{p_0}{\alpha^2} \right) (1 + k)^2,
\]

\[
k = \sum_{j=1}^{N} \frac{k_j}{k_{jo}} = \left( \frac{R_j}{R_{jo}} \right)^3, \quad \rho_0 \left( R_j \frac{d^3R_j}{dt^2} + \frac{3}{2} \left( \frac{dR_j}{dt} \right)^2 \right) = \rho_0 \left( \frac{R_j}{R_{jo}} \right)^{3+} - p
\]

Here \( \rho, p, \) and \( u \) are the averaged density, pressure, and velocity of the particle in the medium and \( k_j \) is the volume concentration of gas for bubbles of radius \( R_j \). The following assumptions were made in the determination of system (0.1) in [3]:

1. the characteristic length \( L \) of the average motion, the average distance \( l \) between the bubbles, and the radius \( R_j \) of the bubbles satisfy the inequalities \( L >> l >> R_j \);
2. nonsphericity of the bubbles and the gas mass can be neglected;
3. the equation of state is written in the acoustic approximation for the liquid component of the medium;
4. the initial values \( k_{jo}, R_{jo}, \) and \( p_0 \) are independent of \( x \).
By comparison with Iordanskii's equations we make one additional assumption here. In the third term of the second equation of (0.1) we omit the term

$$\sum_{j=1}^{N} \rho_{j} k_{j} \left(\frac{dR_{j}}{dt}\right)^{2}$$

in the derivative.

1. We determine the velocity of propagation for small perturbations. Linearizing (0.1) and eliminating the density, we obtain

$$\frac{A}{c_{p}^{2}} \frac{\partial^{2} p}{\partial t^{2}} - \frac{\partial p}{\partial x} - \rho_{0} \sum_{j=1}^{N} k_{j} \frac{\partial}{\partial x} k_{j} = 0,$$

$$\frac{\partial}{\partial t} k_{j} + \Omega_{j}^{2} k_{j} = \Omega_{j}^{2} p \left(\Omega_{j}^{2} = \frac{3 \gamma \rho_{0}}{\rho_{0} R_{j}^{8}}\right).$$

(1.1)

Here $\Omega_{j}$ is the natural frequency of the bubble. We seek the solution in the form

$$p = A e^{i(\omega_{0} - km)}, \quad k_{j} / k_{j} = B_{j} e^{i(\omega_{0} - km)},$$

From (1.1) we easily obtain the following relationship for the phase velocity of sound $c_{2}$:

$$c_{2}^{2} = 1 + \frac{c_{p}^{2}}{c_{p}^{2}} \sum_{j=1}^{N} k_{j} / k_{j} \left(1 - \frac{\omega_{0}^{2}}{\omega_{j}^{2}}\right)^{-1} \left(\frac{c_{j}^{2}}{c_{j}^{2}} = \frac{\gamma \rho_{j}}{\rho_{0} R_{j}^{8}}\right).$$

(1.2)

Here $c_{0}$ is the sound velocity in the liquid and $c_{j}$ is the sound velocity in the medium according to the equilibrium model [1]. Thus, (1.1) describes the motion with dispersion.

With $N \to \infty$ and with the limit conversion in (1.2) we can write

$$c_{2}^{2} = 1 + \frac{c_{p}^{2}}{c_{p}^{2}} \int_{0}^{\infty} \frac{k(R)}{1 - \omega^{2}/\Omega_{j}^{2}} \left(\int_{0}^{\infty} k(R) dR = 1\right).$$

(1.3)

Here $k(R)$ is the fractional concentration of bubbles of a particular kind. The integral in (1.3) for function $k(R)$ of the form

$$\frac{(R/b)^{2}}{1 + (R/b)^{2}},$$

where $b$ is a scale approximating the experimental bubble size distribution [4], can be determined. In this