ON THE THEORY OF PULSE DISCHARGE IN A LIQUID

V. V. Arsent’ev

Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, No. 5, pp. 51–57, 1965

The time dependence of the electric power of underwater discharges
is nearly linear during the first quarter-period. This paper presents
the equations of energy, particle number, and channel expansion
rate under this condition. It is shown that there exists a steady regime
of channel expansion and shock propagation with constant character-
istic properties, and the values of these properties are found.

1. The occurrence of a pulse discharge in a liquid
is accompanied by the penetration of liquid particles
into the arc channel. The channel constitutes a sys-

tem with a variable number of particles. This is indi-
cated by investigations of the underwater explosion
of electrical wires [1], as well as by the fact that
the pressure inside the expanding channel remains
constant for a certain time, the temperature of the
plasma changing insignificantly [2].

The penetration of the liquid particles into the
channel is associated with the heating of the liquid
at the periphery of the channel. This heating is
mainly due to collisions between plasma and liquid
particles; the contribution of radiation and three-par-
ticle recombinations cannot be significant. Due to the
heating there appears between the plasma and the
liquid a gas layer which loses particles to the chan-

nel, where these undergo further heating, dissoci-
ation, and partial ionization.

The rate at which particles penetrate into the
channel is proportional to the rate of energy transfer
by collisions at the periphery of the channel, and is
inversely proportional to the energy of formation of
the gas per particle. The rate of transfer of energy
from the i-th component of the plasma is

$$e_i = \frac{N_i u_i \Delta e_i}{2a}, \quad \Delta e_i = \frac{5n m_i k T}{(m + m_i)^2}. \quad (1.1)$$

Here $N_i$ is the number of particles of the i-th com-
ponent, $u_i$ is their mean thermal speed, $m$ and $m_i$ are
the masses of a liquid molecule and a plasma par-
ticle, respectively, $a$ is the channel radius, and $\Delta e_i$
is the mean amount of energy transferred during one
collision. From (1.1) and the gas-kinetic formula

$$z = \frac{1}{\mu} \frac{\mu N V}{2},$$

which determines the number of colli-
sions of the molecules with a unit area per unit
time, we obtain the rate of penetration of the par-
ticles into the channel

$$N' = \frac{w_e}{\Delta e_i} = 4 \left(\frac{2}{\pi}\right)^{3/2} \frac{1}{(m + m_i)^2} \frac{\Delta e_i}{\Delta e_i}, \quad (1.2)$$

where $q$ is the energy of formation of the gas per
particle.

The theoretical calculation of the coefficient $\gamma$ is
unreliable, since it involves quite arbitrary assump-
tions. However, this coefficient can be found from
any set of experimental data which can be used to
draw a discharge power curve and to determine a
characteristic of the channel. Using the experi-
mental data of Skvortsov et al. [2], we obtain $\gamma = 1/24$.

2. The energy supplied by the electrical circuit
to the underwater spark channel goes towards the
increase of the internal energy of the channel, the
formation of the shock wave, and radiation; the
radiation losses are minor. Analysis of current and
voltage oscillograms of the discharge indicates that
during the first quarter-period the time-dependence
of the power dissipated is linear [3]:

$$w_e = \gamma t. \quad (2.1)$$

At moderate degrees of ionization the mean energy
of a plasma particle is

$$e = \frac{\gamma q k T}{v} + e_d / v,$$

where $v$ is the number of atoms in a molecule of the
liquid and $e_d$ is the dissociation energy per molecule.
The change of the internal energy of the channel per
unit time is then

$$w_i = (N_i)\gamma k T + \frac{\gamma q k T}{v} + N' e_d / v. \quad (2.2)$$

The power transmitted to the shock wave is

$$w_s = \frac{N'k T a'}{V_a} = \frac{2kT a}{\gamma}, \quad (2.3)$$

where $a'$ is the speed of expansion of the channel.
From shock theory it can be shown that one half
of this power is expended in compressing the liquid
and the other half is expended in setting it in mo-
tion. Equations (1.2), (2.1), (2.2), and (2.3) yield

$$\gamma \frac{N'}{\gamma} = \frac{3k v}{2k_d} \frac{(NT)}{\gamma} - \frac{2k v}{e_d} \frac{a'}{\gamma} \frac{(NT)}{\gamma},$$

$$N' = \frac{1}{\gamma} \frac{6}{\gamma} \frac{m k v}{q} \sum_i \frac{v_i m_i v_i}{(m + m_i)^2} \frac{(NT)}{\gamma}. \quad (2.4)$$

The system (2.4) contains three unknown functions
(N,NT, a) and can be closed by the equations of
hydrodynamics.

3. Due to their high nonlinearity, the equations of
hydrodynamics cannot be used here in their general
form, with boundary conditions at the channel boun-
dary and at the shock front. One simplification
is provided by the experimental fact that the speed
of expansion of the channel is constant during the
first quarter-period [2]. The effect of the expanding
channel on the liquid is the same as that of an ex-
panding cylindrical piston. Self-similar problems
involving an expanding piston have been treated by...
several authors [4,6]. In the self-similar problems the transformation to dimensionless variables transforms the equations of hydrodynamics into a system of ordinary differential equations. However, these equations cannot be integrated analytically even in the case of constant piston speed. Thus another simplification is required, for which we shall use the incompressibility of the liquid between the channel and the shock wave. In this region the liquid is compressed by the effect of the shock wave and subsequent density changes may be neglected [2].

Assuming incompressibility, the integration of the hydrodynamic equations yields the pressure field

\[ p = p_0 + \frac{\rho_0}{1 - \frac{a'}{D}} \left[ a'^2 \left( 1 - \frac{a'}{D} \right) + \left( a a'^2 + a^2 \right) \ln \frac{a'}{\rho} \right], \quad (3.1) \]

where \( p_0 \) is the pressure in the channel, \( \rho_0 \) is the density of the undisturbed liquid, and \( R \) is the radial coordinate of the shock front.

The motion of the channel boundary is directly related to the propagation of the shock front. The impulse transmitted by the channel to the surrounding liquid per unit time is equal to the change of momentum of the liquid between the channel and the shock wave. The momentum integral necessarily converges, since it extends only up to the shock front. The equation for the rate of change of momentum is

\[ \frac{d}{dt} \int_0^R a \rho u r dr = 2 \pi \rho_0 u_0, \quad (3.2) \]

where \( u \) is the velocity of the fluid particles. From (3.2) we have

\[ p_0 = \frac{\rho_0}{1 - \frac{a'}{D}} \left[ (aa'^2 + a^3) \frac{1 - a'}{a} + a^2 \frac{1 - a'/D}{a'/D} \right]. \quad (3.3) \]

The solution to the self-similar problem of the motion of a fluid pushed by a uniformly moving piston shows that the shock wave also propagates at a constant velocity and that the pressure at the shock front is constant. Experimental data on shocks generated by underwater explosions also show that the speed of propagation of the shock front is constant when the channel expands at a constant speed [2]. Thus, in (3.1) and (3.3) we may take \( a'^2 = 0 \), \( a/R = a'/D \), so that after expanding the logarithm we obtain

\[ p_f = p_0 + \frac{\rho_0 a'^2}{2} \left( 1 + \frac{a'}{D} \right)^2, \quad p_a = \frac{2 \rho_0 D}{1 + \frac{a'}{D}}. \quad (3.4) \]

The first equation in (3.4) shows that when the channel expands with a constant speed the pressure at the shock wave is lower than that in the channel. From the Rankine-Hugoniot relations and the equation of continuity of an incompressible fluid we obtain

\[ u D = \frac{p_f}{\rho_0}, \quad u = \frac{a a'^2}{R}, \quad or \quad \frac{p_f}{\rho_0} = a'^2. \quad (3.5) \]

Substituting (3.5) into (3.4), we obtain two quadratic equations for the wavefront speed. Equating the coefficients of these equations, we obtain the equation for the speed of expansions of the channel:

\[ a'^4 + \frac{p_0}{a'^3} a^3 - \frac{p_0^3}{a^3} = 0, \quad or \quad a' = 0.8 (\frac{p_0}{\rho_0} - \frac{1}{4}). \]

4. The pressure in the channel can be written in the form

\[ p_a = \frac{b \rho_0 u_0}{a'^2}. \]

Hence, taking account of (3.6) and of the fact that the expansion speed is constant, we obtain

\[ NT = \frac{1.6 \rho_0 u_0 a'^2}{k}. \quad (4.1) \]

Solving (2.4) for \( NT \) and equating the result with (4.1), we obtain an equation for \( a' \):

\[ \gamma \left[ 5k + 2 \frac{a^2}{v} \left( \frac{1}{12} \frac{2}{\pi} \sum \frac{v_i m_i}{\rho_i} \right) \right] = \frac{1.6 \rho_0 u_0 a'^{42}}{k}. \quad (4.2) \]

In the denominator of the left-hand side of (4.2) we can neglect the second term. Then

\[ a' = \left( \frac{1}{\rho_0 \gamma_1} \right)^{\frac{1}{4}}, \quad \left( \gamma_1 = \frac{\gamma}{7} \right). \quad (4.3) \]

Table 1

<table>
<thead>
<tr>
<th>( \gamma_1 )</th>
<th>( a_1 )</th>
<th>( c_1 )</th>
<th>( T_1 )</th>
<th>N particles</th>
<th>( 1 / \mu ) sec</th>
<th>( n_1 ) cm</th>
<th>( P_1 ) kg/cm²</th>
</tr>
</thead>
<tbody>
<tr>
<td>10^11</td>
<td>3</td>
<td>3.3 \times 10^11</td>
<td>188</td>
<td>10000</td>
<td>2.2 \times 10^10</td>
<td>5.4 \times 10^10</td>
<td>400</td>
</tr>
<tr>
<td>10^12</td>
<td>3</td>
<td>3.3 \times 10^10</td>
<td>230</td>
<td>10000</td>
<td>9.3 \times 10^8</td>
<td>4.4 \times 10^10</td>
<td>5400</td>
</tr>
<tr>
<td>10^13</td>
<td>3</td>
<td>3.3 \times 10^9</td>
<td>500</td>
<td>10000</td>
<td>7.2 \times 10^6</td>
<td>7.0 \times 10^8</td>
<td>1500</td>
</tr>
<tr>
<td>10^14</td>
<td>3</td>
<td>3.3 \times 10^8</td>
<td>600</td>
<td>21200</td>
<td>1.3 \times 10^5</td>
<td>9.0 \times 10^8</td>
<td>1700</td>
</tr>
</tbody>
</table>