VIBRATIONAL CREEP OF POLYMER MATERIALS


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In the mechanics of deformed solids it is usually assumed that superposing small amplitude vibrations on a static load has no effect on the over-all characteristics of a material under strain. This hypothesis is reflected in the fact that the existing equations of state for the case of static loads with superposed small vibrations give deformation characteristics which differ little from the corresponding parameters of deformation processes taking place in the absence of excitations. At the same time, substantial changes in the deformation characteristics of a number of materials are observed under certain conditions after the application of alternating stresses of small amplitude. Reports on studies of creep of metals [1, 2], elastomers [3], and concrete [4] have been published, in which the fatigue curves obtained with small vibrations superposed on static loads lie above curves obtained for static loads corresponding to the maximum pulsating load level. Attempts have been made to explain this effect from the standpoint of the molecular-kinetic [5] and phenomenological [5] theories. Certain theoretical considerations and experimental data, discussed in this article, show that superposing a small dynamic component on a static load leads to an increase in the rate of creep of several polymer materials. This effect, which is due mainly to an increase in the polymer temperature as a result of dissipation of vibrational energy, differs from the "vibration effect" observed on elastomers by Slonimskii and Alekseev [3], in which the temperature rise due to the heat generated by vibrations plays no substantial part.

The vibrational creep of polymer materials observed in our experiments differed from that described in [3]. According to results obtained in [3], superposing a vibration component produces the same effect as increasing the static load at the moment indicated by an arrow (Fig. 1a). In our tests, the superposition of vibrations on a static load produces no immediate changes in the behavior of the material; it is only after a certain time (incubation period) that the creep rate begins to increase. When the steady-creepl stage is reached, it is sometimes two to three orders larger than the value recorded before the superposition of vibrations (Fig. 1b); this increase in is accompanied by a substantial increase in specimen temperature. The delay in the acceleration of creep is evidently due to the fact that at the moment of the application of vibrations the material continues to deform elastically, so that the quantity of dissipated energy is small and the increase in specimen temperature slow. In the later stages, heating leads to a more intense energy dissipation which, in turn, generates more heat; as a result, avalanche-type acceleration of the creep rate takes place.

A characteristic feature of the vibrational creep of polymer materials under the conditions studied is that the stress is very small (1-2%) different from the static stress, so that the variation in stress may, in the first approximation, be neglected. The intense heating of a material in vibrational creep must be taken into account, however. The problem of heating of a specimen under a vibrational load was analyzed theoretically by Ratner and Korobov [6] under the assumption that the temperature gradient inside the specimen is negligibly small and that the heat-transfer resistance is concentrated at the specimen surface. The analysis presented below also takes into account the temperature distribution in the specimen interior.

§ 1. It is known that the elasticity of polymer materials is nonequilibrium in character, so that heat evolution during cyclic loading takes place in the entire specimen volume. In the first approximation, for sufficiently high vibration frequencies, the effect of the principal (static) load on heat evolution may be neglected. In this case, the quantity of heat evolved per unit time in a unit volume of a cylindrical specimen in tension is

\[ W = \frac{1}{2} \varepsilon \sigma \ddot{o} J' \]  

(1.1)

Here is the amplitude of the vibration component of tensile stress and is the so-called loss compliance which, for a given material, depends on temperature and frequency . The case of amorphous polymers, this relation is represented by universal curves for the principal (static) load on heat evolution may be neglected. In this case, the quantity of heat evolved per unit time in a unit volume of a cylindrical specimen in tension is

Here and are constants, and is the reference temperature. Experiment shows that, in the interval of practical interest, is linearly dependent on since one can obtain from (1.2)

\[ J' = \frac{K}{\omega^n} \exp \left( \frac{\beta (T - T_0)}{1 - (T - T_0)/c^2} \right) \]  

(1.2)

where , , and are constants. At small , this relation may be approximately represented by

\[ J' = \frac{K}{\omega^n} \exp \left[ \frac{\beta (T - T_0)}{1 - (T - T_0)/c^2} \right] \]  

(1.3)

Thus, for instance, using data from [7], one can show that the following relation is applicable to a plasticized poly-n-butylmethacrylate:

\[ J' = \frac{5 \times 10^4}{\omega^{0.5}} \exp \left[ 0.26 (T - T_0) \right] \]  

(1.4)

here temperature is in K, frequency in cps, J' in cm²/dyne, and is 227°K is the glass transition temperature. The Williams-Landell-Furry equation is not directly applicable to crystalline polymers, including caprolite which is the subject of the present
investigation; there is, however, reason to suppose that the relation (1.3) is applicable to these materials in a certain range of practical interest.

Using Eqs. (1.1) and (1.3) and neglecting the temperature variation in the axial direction, we obtain the following equation for the heat balance of a cylindrical specimen:

$$\rho c \frac{dT}{dt} = \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) + \frac{K}{2} e^{\alpha} \omega^{-\eta} \exp \left[ \frac{\beta (T - T_0)}{\theta} \right].$$

Here $\rho$ is density, $c$ specific heat, $\lambda$ thermal conductivity, and $r$ instantaneous radius (measured from the specimen axis).

With a view to simplifying the analysis of the problem, we neglect both the temperature dependence of density, specific heat and thermal conductivity and the possible variation of stress amplitude $\sigma_0$ along the radius, associated with the variation in the elastic characteristics of the material; the latter is permissible, since the elastic modulus is relatively constant in the temperature range under consideration. In this way, the determination of the temperature variation along the radius and with time is reduced to solving Eq. (1.5) for the real initial and limiting conditions

$$\frac{\partial T}{\partial r} + \frac{\sigma}{\lambda} (T - T_0) = 0 \quad \text{at } r = r_0, \quad T = T_0 \quad \text{at } t = 0.$$ (1.6)

on the assumption that the heat radiation condition is satisfied at the specimen surface; ($\alpha$—heat transfer coefficient; $r_0$—specimen radius).

Analysis of this boundary-value problem is similar to the analysis of stationary thermal explosion, first studied by Frank-Kamenetskii [9] (see also [10]). As is known, this problem has a certain nontrivial singularity, which has an interesting interpretation in the case of heat generated by $\chi \cdot \cdot$ action. To elucidate this singularity, it is advantageous to reduce Eq. (1.5) to dimensionless form; assuming

$$\beta (T - T_0) = u, \quad \tau = \frac{mt}{\varrho_0},$$

$$\xi = \frac{r}{\varrho_0}, \quad \delta = \frac{\beta}{\xi} \varrho_0 \omega^{-\eta} \varrho_0^2,$$

we have

$$\frac{\partial u}{\partial \tau} = \frac{1}{\xi} \frac{\partial}{\partial \xi} \left( \frac{\partial u}{\partial \xi} \right) + \delta e^u. \quad (1.7)$$

Assuming that the limiting conditions are stationary, it may be postulated that at $\tau \gg 1$, i.e., $t \gg \varrho_0^2/\xi$, the solution should approach a steady-state solution which satisfies the equation

$$\frac{1}{\xi} d \frac{d u}{d \xi} + \delta e^u = 0 \quad (1.8)$$

and the boundary condition

$$\frac{d u}{d \xi} + 0 u = 0 \quad \text{at } \xi = 1.$$ (1.9)

The above-mentioned singularity of the problem consists in that the steady-state solution exists not for any values of $\delta$, but only for $\delta \leq \delta_0$, when (as was shown in [11])

$$\delta_0 = \frac{8 \varsigma}{(1 + \varsigma)} \exp \left[ - \frac{4 \varsigma}{\theta (1 + \varsigma)} \right],$$

$$\varsigma = \frac{1}{2} \left( \sqrt{1 + \frac{1}{4} \theta^2} - 1 \right), \quad (1.10)$$

i.e., for

$$r_0^2 \leq \frac{2 \varrho_0 \omega^{-\eta} \varrho_0}{\beta K \delta_0^2} = r_0^2.$$

(1.11)

Fig. 1. a) Effect of vibrations observed in [3]; b) effect of vibrations due to heat evolution. 1) Creep curve (elastic aftereffect) of a polymer material in the absence of vibrations; 2) creep curve (elastic aftereffect) after the superposition of a vibration component. (The moment of application of vibrations is indicated by an arrow.)

In a special case $\delta_0 = 2$, if at the boundary $r = r_0$, $T = T_0$, i.e., $\alpha = \varrho = \infty$, Inequality (1.11) shows that steady-state distribution of temperature is impossible when the specimen radius is larger than $r_*$ In this case, the specimen temperature will rise until the onset of thermal degradation, viscous flow, or other processes of polymer destruction. At $r_0 < r_*$ two steady-state solutions are possible (see, e.g., [10]), one of which (corresponding to more intense heating) is unstable; the latter fact was rigorously substantiated in [12]. At $r_0 = r_*$, the two solutions become one, and the distribution of reduced temperature $u$ along the specimen radius is given by

$$u = \ln \left( \frac{8 \varsigma}{\delta_0} \right) - 2 \ln \left( 1 + \varsigma^2 \right), \quad (1.12)$$

so that the maximum reduced temperature $u_*$ and the reduced temperature at the specimen surface $u_{**}$ are equal to, respectively,

$$u_* = \ln \frac{8 \varsigma}{\delta_0},$$

$$u_{**} = \ln \frac{8 \varsigma}{\delta_0 (1 + \varsigma^2)} - 2 \ln \frac{1}{\theta (1 + \varsigma)} \quad (1.13)$$

If, following [6], one starts by averaging the temperature along the specimen radius and by assuming that all the heat-transfer resistance is concentrated at the boundary, the equation of the heat balance for a cylindrical rod becomes

$$\frac{d T}{d t} = \frac{\varrho_0 \omega^{-\eta} K}{2 \varrho c} \exp \left[ \frac{\beta (T - T_0)}{\theta} \right] - \frac{2 \gamma}{\varrho c \varrho_0} (T - T_0). \quad (1.14)$$