REACTIVE IMPULSE AT HIGH IMPACT VELOCITIES

E. I. Adriankin

Zhurnal prikladnoi mekhaniki i tekhnicheskoi fiziki, No. 1, pp. 88-92, 1965

When a gas moving at high velocity \( v \) impacts against a wall, its temperature rises sharply and radiation begins to play an important part in the energy transfer process [1]. After compression by the shock wave the bulk of the gas begins to undergo radiative cooling, which reduces the reactive impulse transmitted to the obstacle. This effect may occur, for example, when low-density meteoric particles strike a solid wall.

In view of the complexity of the phenomenon, we shall consider the special case where the impacting mass is a plane layer of matter of thickness \( h \) and density \( \rho_0 \). We shall start by assuming that the impact is against a rigid obstacle in a vacuum.

Using the laws of conservation at a strong shock front, we find that during time \( t_1 \) the layer of gas is compressed to a thickness \( h_1 \), while its density \( \rho_1 \), pressure \( p_1 \), and temperature \( T_1 \) are given by the formulas:

\[
\rho_1 = \frac{\gamma+1}{\gamma-1} \rho_0, \quad \rho_1 = \frac{\gamma+1}{2} \rho_0 \rho_0, \quad T_1 = \frac{\gamma^2}{2c_v},
\]

where \( \gamma = \frac{c_p}{c_v} \).

We shall assume that the specific heat of unit mass of the gas \( c_v \) and the ratio of specific heats are constant.

From (1) it follows that in time \( t_1 \) the gas transmits to the wall an impulse \( I_1 = \rho_0 h_1 v \). After compression by the shock wave, the layer of gas begins to expand into the vacuum. The corresponding reactive impulse \( I_2 = I - I_0 \) (here \( I \) is the total impulse transmitted to the wall), disregarding radiation, can be computed starting from the exact solution of the equations of gas dynamics [2]:

\[
I_2 = \xi(g)I_0(\xi(1) = 0.796, \xi(1.4) = 0.825, \xi(3) = 0.865).
\]

The coefficient \( \xi \) characterizes the elasticity of the impact and its value, close to unity\(^a\), shows that expansion usually occurs under conditions closer to elastic than inelastic impact. The radiation sharply reduces \( \xi \), i.e., inelastic conditions are approached.

The rigorous computation of the energy flux from the heated layer of gas requires the solution of the kinetic equation. However, in estimating the effect of radiation on the reactive impulse, it is possible to confine oneself to a consideration of the extreme case where the path length of the quanta is small \( l(\rho_0 T_1) \ll h_1 \). This assumption is admissible, since it turns out that the radiation time is much less than the characteristic time of expansion of the plasma into a vacuum, and the result is relatively independent of the method of de-excitation. We shall write the expression for the reactive impulse in the form:

\[
I_2 = \int \left[ p \left( 0, t \right) \right] dt + \rho_1 R I_1 \left( T_0 - T_1 \right) dt + \rho_1 R I_1 \left( T_1 - T_0 \right) dt + \rho_1 R I_1 \left( T_1 - T_0 \right) dt,
\]

where \( t_1 \) is the time during which it is necessary to take the radiation into account, and \( T_1 \) is the temperature at which radiation becomes important.

For \( l \ll h_1 \) the cooling process is described by the diffusion approximation for the kinetic equation, and the heat flux is expressed in terms of the radiation density gradient. For simplicity, we shall assume that the radiation density is from the outset close to the equilibrium value. Initially, even for \( I \ll h_1 \), the radiation is nonequilibrium owing to the

\(^a\)Physically, the difference between \( \xi \) and unity may be attributed to the redistribution of energy among the gas particles in the nonstationary process.
initial conditions, but it rapidly approaches the equilibrium value and even by the time the free face of the plasma has
cooled to \(0.8 T_1\) the discrepancy is small \([3]\). At this stage the cooling of the gas is described by the equation of radia-
tive heat transfer:

\[
\frac{\partial \rho c_v T}{\partial t} = - \frac{\partial q}{\partial x} - \frac{1}{3} \frac{\partial \epsilon \partial T^4}{\partial x} \quad l = b_0 n T_1^{m/3} \quad \sigma = \frac{5.67 \times 10^{-8}}{\text{cm}^2 \cdot \text{sec} \cdot \text{deg}^4} \quad (3)
\]

where \(l\) is the radiation path length averaged according to Rosseland and given in the form of an interpolation formula.
The boundary conditions for Eq. (3) are:

\[
q(0, t) = 0, \quad q(h_1, t) = 2 \sigma T^4(h_1, t) \leq \sigma T^4(0, t) \quad (4)
\]

The last condition follows from the diffusion relation

\[
q = 2 \sigma T^4 - l \frac{\partial q}{\partial x} \quad \text{for} \quad l \to 0
\]

and expresses the requirement that the kinetic flux at the boundary with the vacuum be equal to half the diffusion flux.

Using the method of moments \([5]\), we can obtain a convenient formula linking the outgoing radiation flux \(q(h_1, t)\)
and the temperature at the wall \(T(0, t)\) (Fig. 1). Equation (3) is equivalent to an infinite number of integral relations ob-
tained by multiplying (3) by \(x^m\) \((m = 0, 1, 2, \ldots)\) and integrating the expressions obtained with respect to \(x\) from 0 to \(h_1\).

We shall satisfy (3) approximately, confining ourselves to the two relations for \(m = 0\) and \(m = 1\):

\[
\frac{d}{dt} \int_0^{h_1} \rho c_v T dx = - q(h_1, t) \quad (5)
\]

\[
\frac{d}{dt} \int_0^{h_1} \rho c_v T x dx = - q(h_1, t) + \frac{16 \sigma b_0 n}{3k} \left[ T^k(0, t) - T^k(h_1, t) \right] (k = \omega + 4) \quad (6)
\]

To obtain an approximate solution of Eqs. (4)-(6), we shall use the property of intense heat transfer that makes the
temperature distribution close to a "plateau" \([6, 7]\). In this case, in the integrand we can replace \(T\) with \(T_0(t) \approx T(0, t)\):

\[
q(h_1, t) = B \frac{4}{h_1} \sigma T_0^4(t) \quad B = \frac{32}{3k} \quad l_0 = b_0 n T_0^{m/3} \quad (7)
\]

As V. P. Buzdin has shown, it is somewhat more correct to give the temperature distribution in the form:

\[
T = T_0(t) \left[ 1 - \left( \frac{x}{h_1} \right)^{3/3k-1} \right] \quad (8)
\]

analogous to that which follows from the solution of \([5]\). In this case

\[
B_1 = \frac{32}{3k5}, \quad \delta = 2 - \frac{\Gamma(1.5 + \alpha)}{\Gamma(\alpha + 1) \Gamma(\alpha + 1) \Gamma(1.5)} \quad \alpha = \frac{1}{k - 1} \quad (9)
\]

From (9) it follows that at large \(k\) the value of \(B_1\) is close to that of \(B\).

It is interesting to note that the temperature distribution in the self-similar problem \([8]\) will also be the exact solution
of the nonself-similar problem of the propagation of a thermal wave from a source with allowance for radiation of energy
from the front \([7]\). For this problem, which physically is similar to the problem of the cooling of a finite volume of gas,
the temperature distribution is given by the solution of \([8]\), which satisfies Eq. (3). This solution must be cut off at a dis-
tance \(x_1(t)\) determined by the boundary condition expressing the energy balance at the wave front

\[
\rho c_v \frac{dx_1}{dt} + \left[ \frac{1}{3} \frac{\partial \epsilon \partial T^4}{\partial x} \right]_{x=x_1} = S[T(x_1, t)] \quad (10)
\]