MULTIPHONON RENORMALIZATION OF THE SPECTRUM OF TWO-DIMENSIONAL EXCITONS IN A MAGNETIC FIELD

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The renormalization of the dispersion law for two-dimensional excitons in a magnetic field is studied by the Green's-function method taking into account multiphonon processes.

The experimental realization of two-dimensional systems [1, 2] has stimulated new theoretical studies along these lines. One of the interesting problems, studied in detail in recent years, is the behavior of excitons in different two-dimensional and quasi-two-dimensional structures exposed to external fields. Thus in [3] the spectrum of Wannier-Mott excitons in strong magnetic fields was studied neglecting the phonon subsystem. In [4, 5] the renormalization of the spectrum of diamagnetic two-dimensional excitons by their interaction with optical phonons at $T = 0$ was studied. The specific nature of the calculation carried out in the last works cited makes it impossible to extrapolate the results obtained there to magnetic field intensities for which the energy corresponding to the cyclotron frequency is less than the excitonic rydberg. The purpose of this work is to study this problem.

Since a magnetic field alters the internal state of an exciton, this changes the binding force between quasiparticles and phonons, intensifying or weakening it depending on the parameters of the crystal. We shall therefore study, by means of the Green's function method developed in [6], the renormalization of the quasiparticle spectrum by optical phonons without assuming that the interaction is weak in the sense of perturbation theory.

In studying a two-dimensional exciton in weak magnetic field \( \frac{\hbar c H}{\mu c^2} < \frac{2 \mu e^4}{\hbar^2 \varepsilon_{so}} \), we shall assume that its energy levels are far apart so that we can neglect scattering between levels. The Hamiltonian of the system in the second quantization representation is derived from first principles. The analytical form of the coupling function is found by the same method as that employed in [7] for the three-dimensional case. Now, however, the interaction of an electron and a hole with two-dimensional optical phonons is taken into account [5], while the transition to the second-quantized representation is carried out for wave functions of the two-dimensional exciton [8] in a magnetic field. As a result the Hamiltonian operator of the system has the typical form [9, 10]

\[
\mathcal{H} = \sum_{k} \varepsilon_H (k) a_k^+ a_k + \sum_{q} \Omega (q) (b_q^+ b_q + 1) + 2 \sum_{k} \xi (H; q) a_{k+q}^+ a_{k} (b_q + b_k^+).
\]

Here

\[
\varepsilon_H (k) = \varepsilon - 2 \mu e^4 \hbar^2 \varepsilon_{so} + \Delta_H + \hbar \pi k^2 / 2 m
\]

is the dispersion law for an exciton in a magnetic field ($\varepsilon$ is the gap width);

\[
\Delta_H = \frac{3 \varepsilon^2 a_{ex}^2}{64 \mu c^2} H^2
\]

is the purely field-induced shift of an energy level (neglecting phonons).

The coupling function between the "magnetic" exciton and the phonons is represented in the form of two terms

\[
\varphi (H; q) = \varphi (q) + \eta_H (q).
\]
The parameters

\[ \varphi_H(q) = \sqrt{\frac{\pi \varepsilon^2 \Omega}{a S q}} \sum_{\rho=1}^{2} \frac{(-1)^\rho}{(1 + \beta q^2)^{3/2}} \]

characterizes the interaction of an exciton with phonons without the field and, up to a factor, is identical to that obtained in [11], while the second term can be regarded as a correction to the interaction owing to the field

\[ \varphi_H(q) = \sqrt{\frac{\pi \varepsilon^2 \Omega}{a S q}} \sum_{\rho=1}^{2} \frac{(-1)^\rho}{(1 + \beta q^2)^{3/2}} \frac{(2 \varepsilon q^2 - 1)}{\varepsilon_0 (1 + \varepsilon_0)} \]

The parameters

\[ d = 2.2 \cdot 10^{-3} (\hbar \omega_c a \varepsilon / \mu e c); \quad p = 1; \quad 2 = h; \quad \varepsilon_0 = (m_0; e a \varepsilon / 4 m)^2, \]

where \( \varepsilon \) is the lattice constant and \( S \) is the "two-dimensional" volume (area) of the main region of the crystal, while the rest of the notation is standard, have been introduced in the formulas (5) and (6).

The optical phonons are assumed to be nondispersive:

\[ \Omega(q) = \Omega. \]

To study the renormalization of the spectrum of a "magnetic" exciton by polarization phonons in the crystal it is necessary to find the mass operator corresponding to the Green's function. Such a mass operator was determined in [6] taking into account the arbitrary magnitude of the coupling with a Hamiltonian of the type (1). In our case it can be written in the form

\[ M^\mu(\kappa) = \sum_{q_i E_{\kappa-q_i} \varphi (H; q_i)^2 A_1(\kappa, q_i)} \frac{|\varphi (H; q_i)|^2 A_1(\kappa, q_i)}{|\varphi (H; q_i)|^2 A_2(\kappa, q_i, q_i)} \]

Here

\[ E_{\kappa} q_i = \omega h - \varepsilon_0 \left[ \kappa - \sum_{i=1}^{n} q_i \right] - \sum_{i=1}^{n} \Omega q_i. \]

The form of the coefficients \( A_n(\kappa, q_1, q_2, ..., q_n) \) is very cumbersome, and since they were determined exactly in [6] we shall not present them here. Since the functions \( |\varphi (H, q_n)|^2 \) have sharp maxima for the same values of \( q_n = q_{n; \kappa} \), this makes it possible, by virtue of the properties of the coefficients \( A_n \) [6], to set